

Coherent Superposition of Mueller-Jones States: Synthesizing Mueller Matrices with an Ellipsometer

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SPECTROSCOPIC ELLIPSOMETRY FOCUS TOPIC



Young's double slit experiment





Young's double slit experiment



No interference pattern! But still there is a coherent superposition























Outline

- 1. A new formalism for coherence and polarization
- Ellipsometry (polarimetry) analysis of Young's double slit
 experiment
- 3. Synthesis of optical components and applications



Consider a Mueller matrix, **M**. Mathematically it is useful to transform **M** into a Hermitian 4x4 matrix **H**, defined as follows:

$$\mathbf{H} = \frac{1}{4} \sum_{i,j=0}^{3} M_{ij} \Pi_{ij}$$

$$\Pi_{ij} = \mathbf{A}(\boldsymbol{\sigma}_i \otimes \boldsymbol{\sigma}_j^*) \mathbf{A}^{-1}$$

$$\sigma_{0} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_{1} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & i & -i & 0 \end{pmatrix}$$

S. R. Cloude, Optik 75, 26–36 (1986)



Explicitely:

$\mathbf{H} = \frac{1}{4}$	$\begin{pmatrix} M_{00} + M_{11} \\ M_{22} + M_{33} \end{pmatrix}$	$M_{01} + M_{10} -i(M_{23} - M_{32})$	$M_{02} + M_{20} + i(M_{13} - M_{31})$	$ \begin{array}{c} M_{03} + M_{30} \\ -i(M_{12} - M_{21}) \end{array} $
	$ \frac{M_{01} + M_{10}}{+i(M_{23} - M_{32})} $	$M_{00} + M_{11}$ - $M_{22} - M_{33}$	$M_{12} + M_{21} + i(M_{03} - M_{30})$	$M_{13} + M_{31}$ $-i(M_{02} - M_{20})$
	$\frac{M_{02} + M_{20}}{-i(M_{13} - M_{31})}$	$M_{12} + M_{21}$ $-i(M_{03} - M_{30})$	$M_{00} - M_{11} + M_{22} - M_{33}$	$M_{23} + M_{32} + i(M_{01} - M_{10})$
	$\frac{M_{03} + M_{30}}{M_{12} - M_{21}}$		$\frac{M_{23} + M_{32}}{-i(M_{01} - M_{10})}$	$ \begin{array}{c} $



S. R. Cloude, Optik 75, 26–36 (1986)

If and only if **M** is nondepolarizing, **H** has <u>Rank 1</u> and can be rewritten as.

 $\mathbf{H} = |b\rangle \langle b|$

 $|b\rangle$ is the eigenvector of **H** corresponding to its single non-zero eigenvalue

If
$$|b\rangle = \begin{pmatrix} \tau \\ \alpha \\ \beta \\ \gamma \end{pmatrix}$$
 then $\mathbf{H} = |b\rangle\langle b| = \begin{pmatrix} \tau\tau^* & \tau\alpha^* & \tau\beta^* & \tau\gamma^* \\ \alpha\tau^* & \alpha\alpha^* & \alpha\beta^* & \alpha\gamma^* \\ \beta\tau^* & \beta\alpha^* & \beta\beta^* & \beta\gamma^* \\ \gamma\tau^* & \gamma\alpha^* & \gamma\beta^* & \gamma\gamma^* \end{pmatrix}$

E. Kuntman et al., J. Opt. Soc. Am. A 34, 80-86 (2017)



To study coherence and partial coherence it is very interesting to define still a new 4x4 complex matrix :

$$\mathbf{Z} = \begin{pmatrix} \tau & \alpha & \beta & \gamma \\ \alpha & \tau & -i\gamma & i\beta \\ \beta & i\gamma & \tau & -i\alpha \\ \gamma & -i\beta & i\alpha & \tau \end{pmatrix}$$

$$\mathbf{M}_J = \mathbf{Z}\mathbf{Z}^* = \mathbf{Z}^*\mathbf{Z}$$

The **Z** matrix is a complex matrix state that, when multiplied with its complex conjugate, gives a real valued Mueller-Jones matrix with elements that are observable quantities in experimental polarization optics

Analogy with a quantum mechanical wave function, ψ



Superposition of equal states: interference



A composition of two states:

 $\psi = a\psi_a + b\psi_b$

that are equal except for a phase $\psi_b = e^{i\phi}\psi_a$ $a = b = \frac{1}{\sqrt{2}}$

$$\psi\psi^* = \frac{1}{2}\psi_a\psi_a^*(1+e^{i\phi})(1+e^{-i\phi})$$

= $\psi_a\psi_a^*(1+\cos\phi).$



Superposition of different Mueller-Jones states



Equivalent descriptions

 $\mathbf{J} = a\mathbf{J}_a + b\mathbf{J}_b$ $\mathbf{Z} = a\mathbf{Z}_a + b\mathbf{Z}_b$

 $|h\rangle = a|h_a\rangle + b|h_b\rangle$

In the **Z** matrix description:

$$\mathbf{M}_J = \mathbf{Z}\mathbf{Z}^* = aa^*\mathbf{Z}_a\mathbf{Z}_a^* + bb^*\mathbf{Z}_b\mathbf{Z}_b^* + ab^*\mathbf{Z}_a\mathbf{Z}_b^* + ba^*\mathbf{Z}_b\mathbf{Z}_a^*$$



Two particular examples of Z superpositions



- Same Mueller-Jones states

$$\mathbf{Z} = \frac{1}{\sqrt{2}}\mathbf{Z}_a + \frac{e^{i\phi}}{\sqrt{2}}\mathbf{Z}_a = \frac{1}{\sqrt{2}}\mathbf{Z}_a(1+e^{i\phi})$$

$$\mathbf{M}_{J} = \mathbf{Z}_{a} \mathbf{Z}_{a}^{*} (1 + \cos \phi) = \mathbf{M}_{a} (1 + \cos \phi)$$
Intensity
interference

$$\begin{aligned} \mathbf{Z}_{0} &= \frac{1}{\sqrt{2}} \mathbf{Z}_{H} + \frac{e^{i\phi}}{\sqrt{2}} \mathbf{Z}_{V}, \qquad \mathbf{Z}_{V} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & i \\ 0 & 0 & -i & 1 \end{pmatrix} \qquad \mathbf{Z}_{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -i \\ 0 & 0 & 1 & -i \\ 0 & 0 & i & 1 \end{vmatrix} \\ \mathbf{Z}_{0} &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 + e^{i\phi} & 1 - e^{i\phi} & 0 & 0 \\ 1 - e^{i\phi} & 1 + e^{i\phi} & 0 & 0 \\ 0 & 0 & 1 + e^{i\phi} & -i(1 - e^{i\phi}) \\ 0 & 0 & i(1 - e^{i\phi}) & 1 + e^{i\phi} \end{pmatrix} \\ \mathbf{M}_{0} &= \mathbf{Z}_{0} \mathbf{Z}_{0}^{*} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos \phi & -\sin \phi \\ 0 & 0 & \sin \phi & \cos \phi \end{pmatrix} \qquad \text{(But no interference in the intensity. No visible fringes.)} \end{aligned}$$



Partial coherence and incoherence

At a given time, space, and frequency all phases can be considered as constants, and the linear superposition is coherent. But when **statistical averages** (time average, spatial average, and/or frequency average) that happen during measurement are taken into account the result will be **depolarizing**

Two
states
$$\mathbf{Z} = \frac{1}{\sqrt{2}} \mathbf{Z}_a + \frac{e^{i\phi}}{\sqrt{2}} \mathbf{Z}_b \longrightarrow \mathbf{M}_J = \frac{1}{2} \mathbf{M}_a + \frac{1}{2} \mathbf{M}_b + \frac{e^{-i\phi}}{2} \mathbf{Z}_a \mathbf{Z}_b^* + \frac{e^{i\phi}}{2} \mathbf{Z}_b \mathbf{Z}_a^*$$
$$\mathbf{Z}' = \frac{1}{\sqrt{2}} \mathbf{Z}_a - \frac{e^{i\phi}}{\sqrt{2}} \mathbf{Z}_b \longrightarrow \mathbf{M}'_J = \frac{1}{2} \mathbf{M}_a + \frac{1}{2} \mathbf{M}_b - \frac{e^{-i\phi}}{2} \mathbf{Z}_a \mathbf{Z}_b^* - \frac{e^{i\phi}}{2} \mathbf{Z}_b \mathbf{Z}_a^*$$

For a combined system fluctuating between these two states:

$$\mathbf{M}_{\text{average}} = \frac{1}{2}\mathbf{M}_a + \frac{1}{2}\mathbf{M}_b$$



Partial coherence and incoherence

If the there are phase fluctuations within the measurment conditions: Coherence terms are washed out

$$\mathbf{Z} = \frac{1}{\sqrt{2}}\mathbf{Z}_a + \frac{e^{i\phi}}{\sqrt{2}}\mathbf{Z}_b \longrightarrow \mathbf{M}_J = \frac{1}{2}\mathbf{M}_a + \frac{1}{2}\mathbf{M}_b + \frac{e^{-i\phi}}{2}\mathbf{Z}_a\mathbf{Z}_b^* + \frac{e^{i\phi}}{2}\mathbf{Z}_b\mathbf{Z}_a^*$$

A more general formula for the combination of two states when the phases of the complex coefficients fluctuate:

$$\mathbf{M} = |a|^2 \mathbf{Z}_a \mathbf{Z}_a^* + |b|^2 \mathbf{Z}_b \mathbf{Z}_b^* + \langle ab^* \rangle \mathbf{Z}_a \mathbf{Z}_b^* + \langle ba^* \rangle \mathbf{Z}_b \mathbf{Z}_a^*$$

Coherence index

$$p = \sqrt{\frac{\langle ab^* \rangle \langle ba^* \rangle}{|a|^2 |b|^2}}$$
 p=0 total incoherence p=1 total coherence



Partial coherence and incoherence

The superposition can lead two 3 different situations:

$$\mathbf{M}_J = \mathbf{Z}\mathbf{Z}^* = aa^*\mathbf{Z}_a\mathbf{Z}_a^* + bb^*\mathbf{Z}_b\mathbf{Z}_b^* + ab^*\mathbf{Z}_a\mathbf{Z}_b^* + ba^*\mathbf{Z}_b\mathbf{Z}_a^* \quad \text{Coherence}$$

 $\mathbf{M} = |a|^2 \mathbf{Z}_a \mathbf{Z}_a^* + |b|^2 \mathbf{Z}_b \mathbf{Z}_b^* \qquad \text{Incoherence}$

The same calculus covers all

 $\mathbf{M} = |a|^2 \mathbf{Z}_a \mathbf{Z}_a^* + |b|^2 \mathbf{Z}_b \mathbf{Z}_b^* + \langle ab^* \rangle \mathbf{Z}_a \mathbf{Z}_b^* + \langle ba^* \rangle \mathbf{Z}_b \mathbf{Z}_a^* \quad \text{Partial coherence}$

E. Kuntman et al., Phys. Rev. A **95**, 063819 (2017)



Can we study Young's double slit experiment in a polarimeter or ellipsometer?



Yes, but a detector with some spatial resolution is needed

To be interesting the polarization aspect needs to be considered





F. Arago and A. Fresnel, "Mémoire sur l'action que les rayons de lumière polarisée exercent les uns sur les autres," Ann. Chim. Phys. 10, 288–305 (1819). (derivation of the laws of interference and polarization)



Under common experimental conditions the classic Young's double-slit experiment made with orthogonal polarizers is equivalent to double refraction in a crystal.

R. Ossikovski, et al., "On the equivalence between Young's double-slit and crystal double-refraction interference experiments," J. Opt. Soc. Am. A 34, 1309-1314 (2017)





But using polarizers the interference becomes visible for the naked eye or a camera













O.Arteaga, et al. Opt. Lett. 42, 3900-3903 (2017)



$$\begin{aligned} \mathbf{Z}_{\mathbf{V}} &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & i \\ 0 & 0 & -i & 1 \end{pmatrix} \qquad \mathbf{Z}_{\mathbf{H}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -i \\ 0 & 0 & i & 1 \end{pmatrix} \\ \mathbf{Z}_{0} &= \frac{1}{\sqrt{2}} \mathbf{Z}_{H} + \frac{e^{i\phi}}{\sqrt{2}} \mathbf{Z}_{V} \\ \mathbf{Z}_{0} &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 + e^{i\phi} & 1 - e^{i\phi} & 0 & 0 \\ 1 - e^{i\phi} & 1 + e^{i\phi} & 0 & 0 \\ 0 & 0 & 1 + e^{i\phi} & -i(1 - e^{i\phi}) \\ 0 & 0 & i(1 - e^{i\phi}) & 1 + e^{i\phi} \end{pmatrix} \\ \mathbf{M}_{0} &= \mathbf{Z}_{0} \mathbf{Z}_{0}^{*} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos \phi & -\sin \phi \\ 0 & 0 & \sin \phi & \cos \phi \end{pmatrix} \end{aligned}$$



Other polarimetric configurations





$$\mathbf{Z}_{a} = \frac{1}{\sqrt{2}} \mathbf{Z}_{45} \mathbf{Z}_{H} + \frac{e^{i\phi}}{\sqrt{2}} \mathbf{Z}_{45} \mathbf{Z}_{V} = \mathbf{Z}_{45} \mathbf{Z}_{0}, \qquad \mathbf{Z}_{b} = \frac{1}{\sqrt{2}} \mathbf{Z}_{H} + \frac{e^{i\phi}}{\sqrt{2}} \mathbf{Z}_{R} \mathbf{Z}_{V}, \qquad \mathbf{Z}_{c} = \frac{1}{\sqrt{2}} \mathbf{Z}_{R} \mathbf{Z}_{H} + \frac{e^{i\phi}}{\sqrt{2}} \mathbf{Z}_{R} \mathbf{Z}_{V} = \mathbf{Z}_{R} \mathbf{Z}_{0}, \qquad \mathbf{Z}_{c} = \frac{1}{\sqrt{2}} \mathbf{Z}_{R} \mathbf{Z}_{H} + \frac{e^{i\phi}}{\sqrt{2}} \mathbf{Z}_{R} \mathbf{Z}_{V} = \mathbf{Z}_{R} \mathbf{Z}_{0}, \qquad \mathbf{Z}_{c} = \frac{1}{\sqrt{2}} \mathbf{Z}_{R} \mathbf{Z}_{H} + \frac{e^{i\phi}}{\sqrt{2}} \mathbf{Z}_{R} \mathbf{Z}_{V} = \mathbf{Z}_{R} \mathbf{Z}_{0}, \qquad \mathbf{Z}_{c} = \frac{1}{\sqrt{2}} \mathbf{Z}_{R} \mathbf{Z}_{H} + \frac{e^{i\phi}}{\sqrt{2}} \mathbf{Z}_{R} \mathbf{Z}_{V} = \mathbf{Z}_{R} \mathbf{Z}_{0}, \qquad \mathbf{Z}_{c} = \frac{1}{\sqrt{2}} \mathbf{Z}_{R} \mathbf{Z}_{H} + \frac{e^{i\phi}}{\sqrt{2}} \mathbf{Z}_{R} \mathbf{Z}_{V} = \mathbf{Z}_{R} \mathbf{Z}_{0}, \qquad \mathbf{Z}_{c} = \frac{1}{\sqrt{2}} \mathbf{Z}_{R} \mathbf{Z}_{H} + \frac{e^{i\phi}}{\sqrt{2}} \mathbf{Z}_{R} \mathbf{Z}_{V} = \mathbf{Z}_{R} \mathbf{Z}_{0}, \qquad \mathbf{Z}_{c} = \frac{1}{\sqrt{2}} \mathbf{Z}_{R} \mathbf{Z}_{H} + \frac{e^{i\phi}}{\sqrt{2}} \mathbf{Z}_{R} \mathbf{Z}_{V} = \mathbf{Z}_{R} \mathbf{Z}_{0}, \qquad \mathbf{Z}_{c} = \frac{1}{\sqrt{2}} \mathbf{Z}_{R} \mathbf{Z}_{H} + \frac{e^{i\phi}}{\sqrt{2}} \mathbf{Z}_{R} \mathbf{Z}_{V} = \mathbf{Z}_{R} \mathbf{Z}_{0}, \qquad \mathbf{Z}_{c} = \frac{1}{\sqrt{2}} \mathbf{Z}_{R} \mathbf{Z}_{R} \mathbf{Z}_{H} + \frac{e^{i\phi}}{\sqrt{2}} \mathbf{Z}_{R} \mathbf{Z}_{H} + \frac{e^{i\phi}}{\sqrt{2}} \mathbf{Z}_{R} \mathbf{Z}_{R} \mathbf{Z}_{H} = \mathbf{Z}_{c} \mathbf{Z}_{R} \mathbf{Z$$







Partial coherence

$$\mathbf{M} = |a|^{2} \mathbf{Z}_{a} \mathbf{Z}_{a}^{*} + |b|^{2} \mathbf{Z}_{b} \mathbf{Z}_{b}^{*} + \langle ab^{*} \rangle \mathbf{Z}_{a} \mathbf{Z}_{b}^{*} + \langle ba^{*} \rangle \mathbf{Z}_{b} \mathbf{Z}_{a}^{*}$$
$$\mathbf{M} = \frac{1}{2} \mathbf{M}_{H} + \frac{1}{2} \mathbf{M}_{V} + \frac{\langle e^{-i\phi(x,y)} \rangle}{2} \mathbf{Z}_{V} \mathbf{Z}_{H}^{*} + \frac{\langle e^{i\phi(x,y)} \rangle}{2} \mathbf{Z}_{H} \mathbf{Z}_{V}^{*}$$
$$\langle e^{\pm i\phi(x,y)} \rangle = \frac{1}{A_{P}} \int_{A_{P}} e^{\pm i\phi(x,y)} \mathrm{d}S \qquad A_{P} \text{ is the pinhole area}$$





Basic example. A linear retarder is synthetised "polarizers"

$$\mathbf{M}_{0} = \mathbf{Z}_{0}\mathbf{Z}_{0}^{*} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\phi & -\sin\phi \\ 0 & 0 & \sin\phi & \cos\phi \end{pmatrix}$$

$$\phi = -\pi/2$$
 Horizontal QWP
 $\phi = \pi/2$ Vertical QWP
 $\phi = \pi$ Half Wave Plate



Which experimental conditions allow the synthesis?

1. The detector needs to have enough spatial resolution to resolve the fringes

$$\Delta x = D\lambda/d$$

Δx: distance between two fringesD: distance from the crystal to the detectord: separation between the H and V beams in the crystal

2. The light source needs to be coherent enough

Can only be made with a laser? NO

In a standard polarimetric experiment the required temporal and spatial coherence is determined by the thickness of double refractive crystal

Thicker crystal 💻

Larger separation of H and V beams





From: http://zeiss-campus.magnet.fsu.edu/tutorials/coherence/indexflash.html

Experimental examples of fringe images recorded with a Xe light source





Generation of periodic spatial modulation





Generation of periodic spatial modulation



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As the modulation is periodic, synchronous detection is possible Development of new ellipsometers
 and polarimeters based on spatial phase modulation



Summary



- A new formalism for polarization and coherence has been introduced to study with the same algebraic tools the three regimes: coherence, partial coherence and incoherence.
- In ellipsometry we are familiar with interference effects occuring in the micro- or nano-scale (e.g., thin films), but we are less familiar with superposition processes that can be macroscopically controlled, such in Young experiment
- Well controlled light superposition processes allow the synthesis of polarization components which can be used for future ellipsometry and polarimetry instrumentation



8th Intenational Conference on Spectroscopic Ellipsometry (ICSE-8) Barcelona, 26-31 May 2019 Chair: M. Isabel Alonso









The state of the s