UNIVERSITAT DE BARCELONA

Phase and polarization: from Fresnel-Arago interference Laws to metasurfaces

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Preface

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Preface

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Outline

1. Basic ideas about polarization and phases
2. Fresnel-Arago interference laws
3. Pancharatnam theory of interference
4. Metasurfaces and geometric phase optical elements
5. One more thing (if there is time)

1. Basic ideas about polarization and phases

Phase and polarization: the basics

Maxwell equations admit **plane wave** solutions.

$$\vec{E} = \vec{E}_0 e^{i(\omega t - kz)}$$

$$\vec{E} = \begin{pmatrix} E_{0x} e^{i(\omega t - kz + \phi_x)} \\ E_{0y} e^{i(\omega t - kz + \phi_y)} \\ 0 \end{pmatrix} = \begin{pmatrix} E_{0x} e^{i\phi_x} \\ E_{0y} e^{i\phi_y} \\ 0 \end{pmatrix} e^{i(\omega t - kz)} = \underbrace{\begin{pmatrix} E_{0x} e^{i(\phi_x - \phi_y)} \\ E_{0y} \\ 0 \end{pmatrix}}_{\text{Polarization}} \underbrace{e^{i(\omega t - kz + \phi_y)}}_{\text{Global (overall) phase}}$$

For quasi-monochromatic light.

Polarization experiment usually involves mesurement times of a very high number of cycles so that the common overall phase can be removed

Phase and polarization: the basics

The **Jones vector** is defined as

$$\mathbf{E} = \begin{pmatrix} E_{0x} e^{i\phi_x} \\ E_{0y} e^{i\phi_y} \end{pmatrix} = \underbrace{e^{i\phi_y}}_{\text{Phase}} \underbrace{\begin{pmatrix} E_{0x} e^{i(\phi_x - \phi_y)} \\ E_{0y} \end{pmatrix}}_{\text{3 parameters are enough to describe complete polarization}}$$

- The Jones matrix is a 2x2 complex matrix that can multiply the Jones vector
- Of particular interest for this presentation is the **Jones matrix of a linear retarder** that introduces a different retardance for x and y polarizations.

Phase and polarization: the basics

The Jones matrix of a linear retarder is

$$\mathbf{J} = \begin{pmatrix} e^{-i\delta_x} & 0 \\ 0 & e^{-i\delta_y} \end{pmatrix} = \begin{pmatrix} e^{-i\frac{2\pi}{\lambda}n_x d} & 0 \\ 0 & e^{-i\frac{2\pi}{\lambda}n_y d} \end{pmatrix} = \underbrace{e^{-i\frac{2\pi}{\lambda}n_x d}}_{\text{Phase}} \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{2\pi}{\lambda}(n_x - n_y)d} \end{pmatrix}}_{\text{Polarization transformation}}$$

$$\delta = \frac{2\pi}{\lambda}(n_x - n_y)d$$

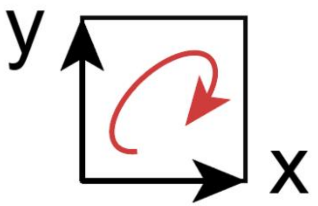
Sometimes this is also written as:

$$\mathbf{J} = e^{-i\frac{2\pi}{\lambda}(n_x + n_y)d} \begin{pmatrix} e^{-i\frac{\pi}{\lambda}(n_x - n_y)d} & 0 \\ 0 & e^{i\frac{\pi}{\lambda}(n_x - n_y)d} \end{pmatrix} = \underbrace{e^{-i\frac{2\pi}{\lambda}(n_x + n_y)d}}_{\text{“Isotropic” Phase}} \underbrace{\begin{pmatrix} e^{-i\delta/2} & 0 \\ 0 & e^{i\delta/2} \end{pmatrix}}_{\text{Polarization transformation}}$$

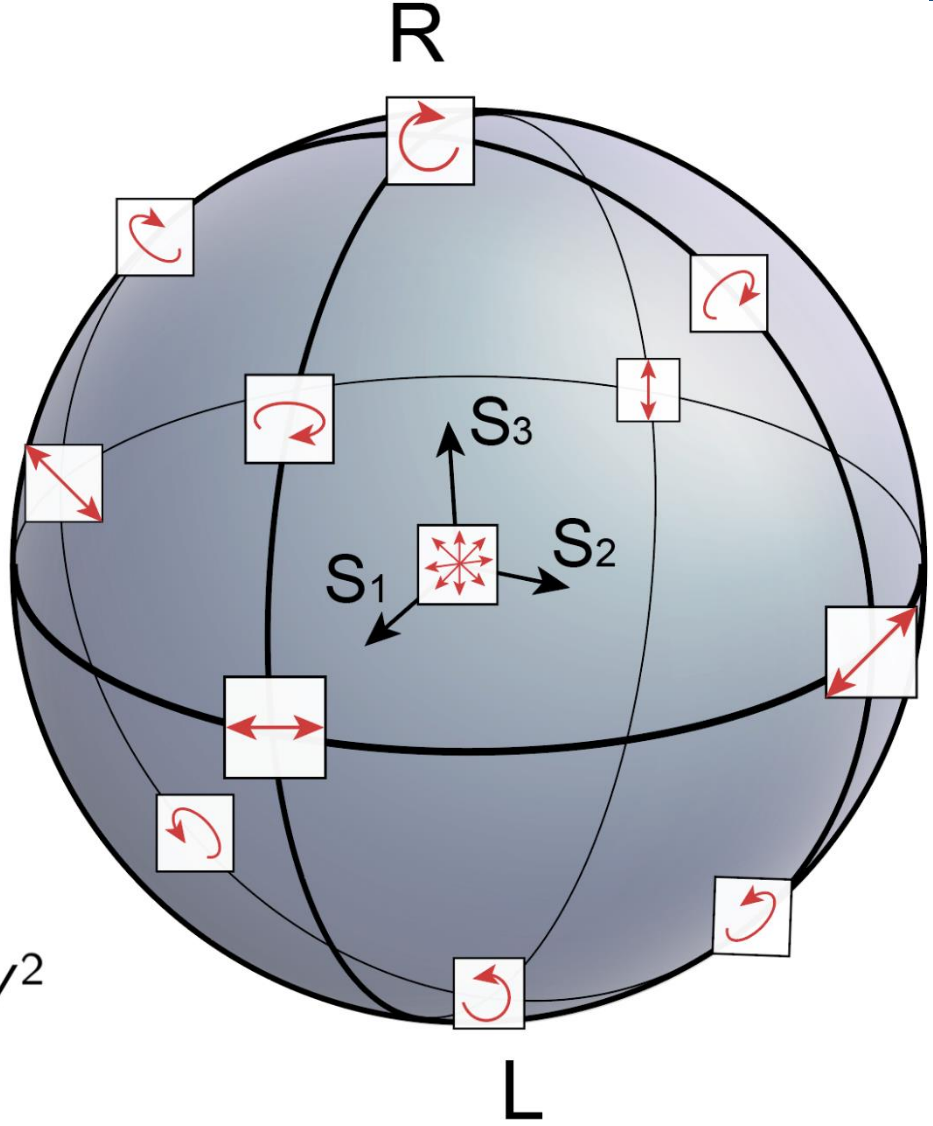
Stokes vectors and Poincaré Sphere

Poincaré sphere:
from Wikipedia

$$S_0 = I = E_x^2 + E_y^2$$
$$S_1 = Q = E_x^2 - E_y^2$$
$$S_2 = U = 2E_x E_y \cos\delta$$
$$S_3 = V = 2E_x E_y \sin\delta$$



$$radius = I^2 \geq Q^2 + U^2 + V^2$$

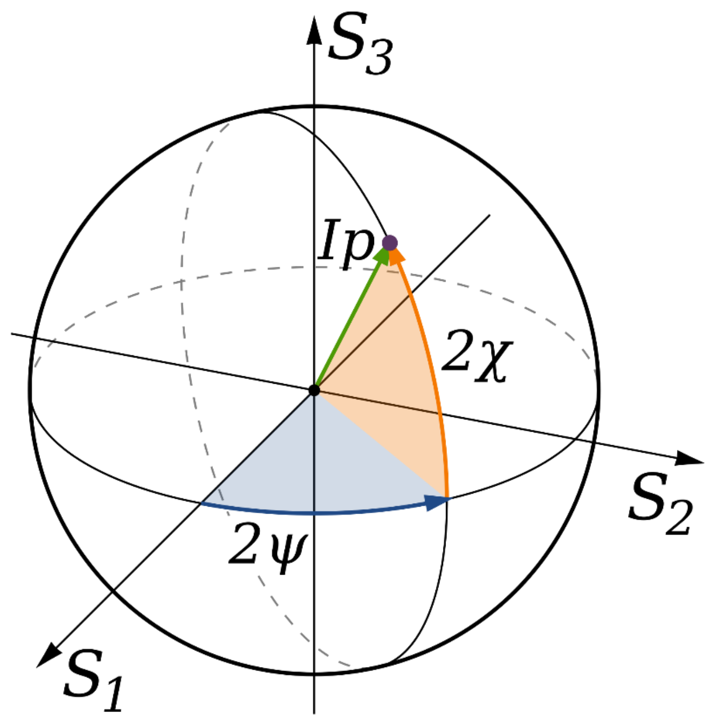


The quantities S_0, S_1, S_2, and S_3 are the observables of the polarized field

Stokes vectors and Poincaré Sphere

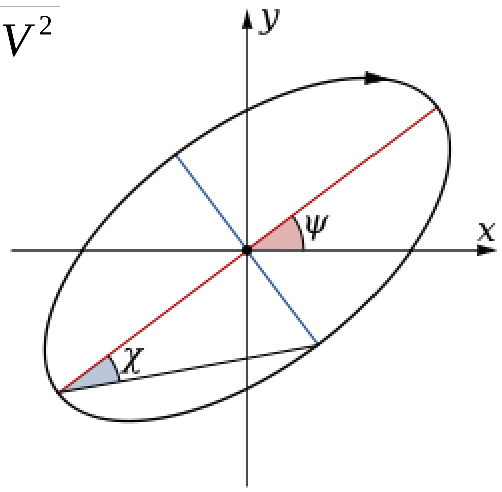
Stokes vectors can also be written in spherical coordinates

$$\mathbf{S} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} I \\ I_x - I_y \\ I_{45} - I_{135} \\ I_+ - I_- \end{bmatrix} = \begin{bmatrix} I \\ Ip\cos(2\psi)\cos(2\chi) \\ Ip\sin(2\psi)\cos(2\chi) \\ Ip\sin(2\chi) \end{bmatrix}$$



No depolarization: $I = \sqrt{Q^2 + U^2 + V^2}$

- I Intensity
- p Degree of polarization
- ψ Azimuth
- χ Ellipticity



Stokes vectors and Poincaré Sphere

Linear transformations of Stokes vectors are described by **Mueller matrices**

$$\mathbf{S}_{out} = \mathbf{M}\mathbf{S}_{in}$$

$$\mathbf{M} = \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \\ m_{30} & m_{31} & m_{32} & m_{33} \end{bmatrix}$$

Phenomenological description of any scattering experiment

Only real numbers!

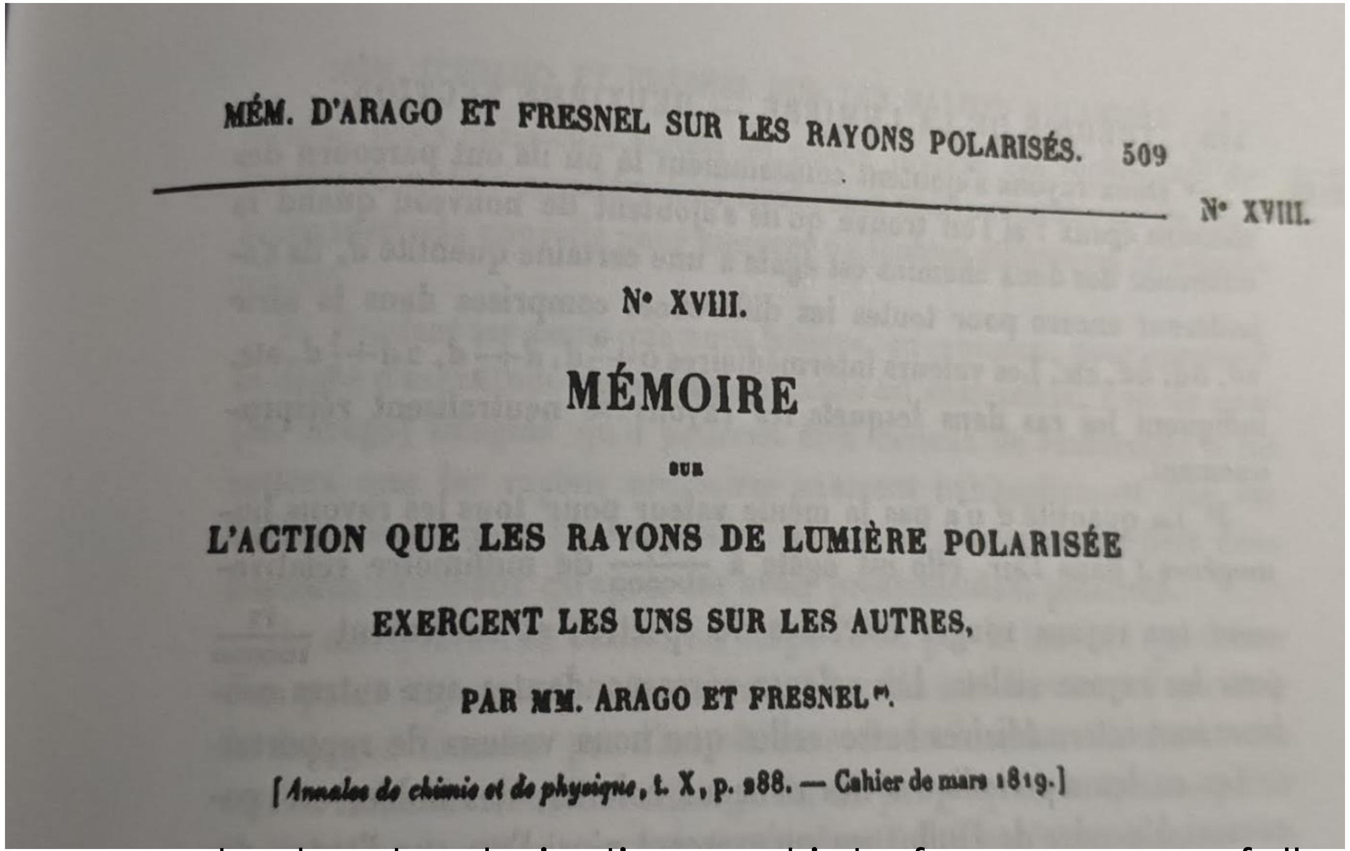
Contains no phase information

2 Fresnel-Arago interference laws

Fresnel-Arago Interference Laws

F. Arago and A. Fresnel, “Mémoire sur l’action que les rayons de lumière polarisée exercent les uns sur les autres,” Ann. Chim. Phys. 10, 288–305 (1819)
(derivation of the laws of interference and polarization)

Augustin Jean Fresnel
1788-1827



MÉM. D'ARAGO ET FRESNEL SUR LES RAYONS POLARISÉS. 509

N° XVIII.

N° XVIII.

MÉMOIRE

SUR

L'ACTION QUE LES RAYONS DE LUMIÈRE POLARISÉE

EXERCENT LES UNS SUR LES AUTRES,

PAR MM. ARAGO ET FRESNEL (1).

[Annales de chimie et de physique, t. X, p. 288. — Cahier de mars 1819.]

Francesc Aragó (François Arago) 1786-1853

Much of the concepts about polarization and interference were fully described before electromagnetic theory was developed

Fresnel-Arago Interference Laws

nissent aussi trois systèmes de franges; mais ils sont séparés des premiers. Si, tout restant dans le même état, on substitue maintenant au rhomboïde une lame de sulfate de chaux ou de cristal de roche qui ne donne pas deux images distinctes, les six systèmes, au lieu d'en produire trois par leur superposition, se réduisent à celui du milieu. Ce résultat remarquable démontre : 1° que les franges résultantes de l'interférence des rayons ordinaires sont complémentaires des franges produites par les rayons extraordinaires; et 2° que ces deux systèmes sont mutuellement disposés de manière qu'une frange brillante du premier système correspond à une frange obscure du second, et réciproquement ; sans ces deux conditions, on n'apercevrait autre chose qu'une lumière uniforme et continue sur les deux côtés des franges centrales. On retrouve donc ici la différence d'une demi-ondulation, comme dans le phénomène des anneaux colorés.

Les expériences que nous venons de rapporter conduisent donc en définitive aux conséquences suivantes :

1° Dans les mêmes circonstances où deux rayons de lumière ordinaire paraissent mutuellement se détruire, deux rayons polarisés à angles droits ou en sens contraires n'exercent l'un sur l'autre aucune action appréciable;

2° Les rayons de lumière polarisés dans un seul sens agissent l'un sur l'autre comme les rayons naturels : en sorte que, dans ces deux espèces de lumières, les phénomènes d'interférence sont absolument les mêmes;

3° Deux rayons primitivement polarisés en sens contraires peuvent ensuite être ramenés à un même plan de

polarisation, sans néanmoins acquérir par là la faculté de s'influencer;

4° Deux rayons polarisés en sens contraires, et ramenés à des polarisations analogues, s'influencent comme les rayons naturels, s'ils proviennent d'un faisceau primitivement polarisé dans un seul sens;

5° Dans les phénomènes d'interférence produits par des rayons qui ont éprouvé la double réfraction, la place des franges n'est pas déterminée uniquement par la différence des chemins et par celle des vitesses; et dans quelques circonstances que nous avons indiquées, il faut tenir compte, de plus, d'une différence égale à une demi-ondulation.

Toutes ces lois se déduisent, comme on l'a vu, d'expériences directes. On pourrait y arriver plus simplement encore en partant des phénomènes que présentent les lames cristallisées; mais il faudrait déjà admettre que les teintes dont ces lames se colorent quand on les éclaire par un faisceau de lumière polarisée, résultent de l'interférence de plusieurs systèmes d'ondes. Les démonstrations que nous avons rapportées ont l'avantage d'établir les mêmes lois, indépendamment de toute hypothèse.

Fresnel-Arago Interference Laws

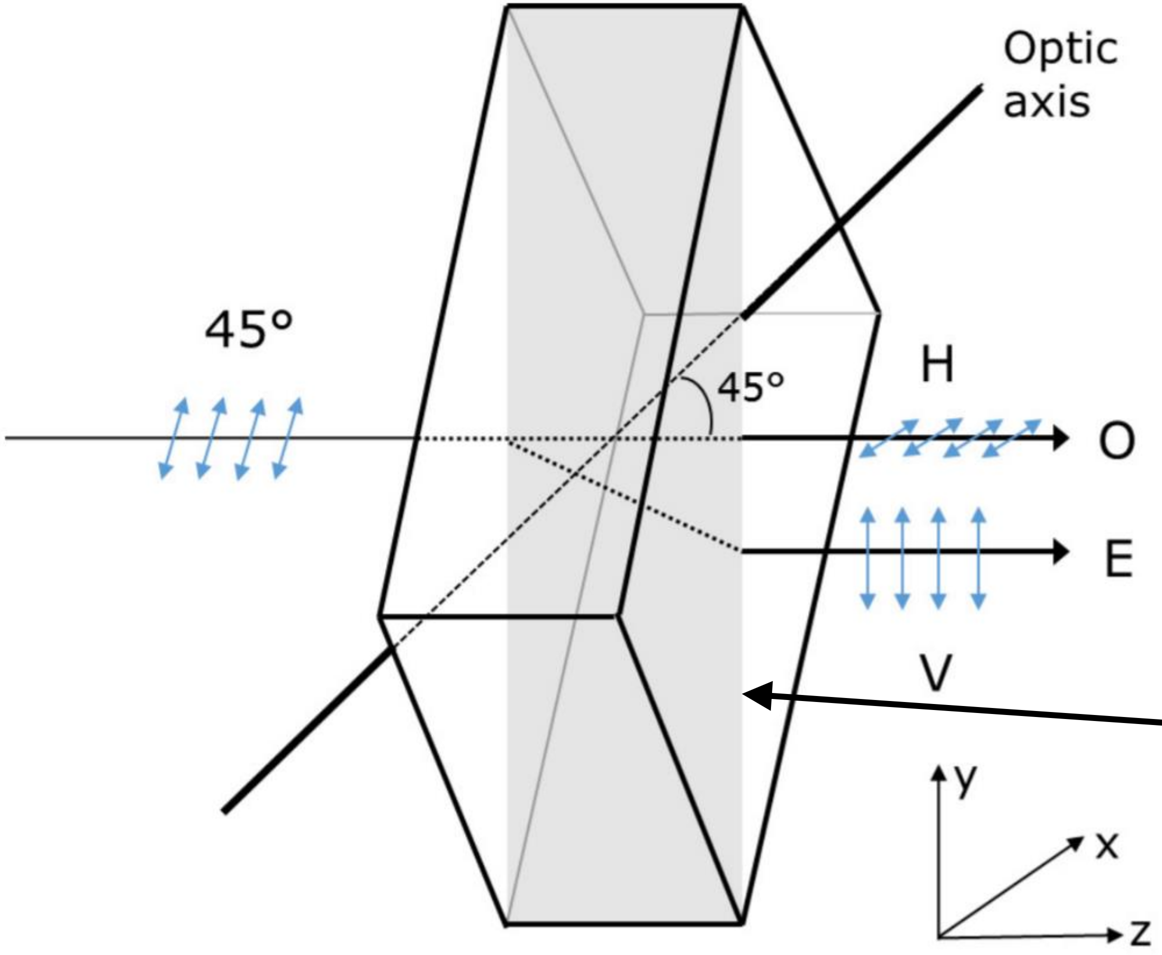
1. Two linearly polarized light waves of orthogonal polarizations do not interfere.
2. Two linearly polarized light waves of identical polarization interfere.
3. Two linearly polarized light waves of orthogonal polarizations obtained from a natural (unpolarized) light source do not interfere if brought to common linear polarization.
4. Two linearly polarized light waves of orthogonal polarizations obtained from a linearly polarized light source interfere if brought to common linear polarization.

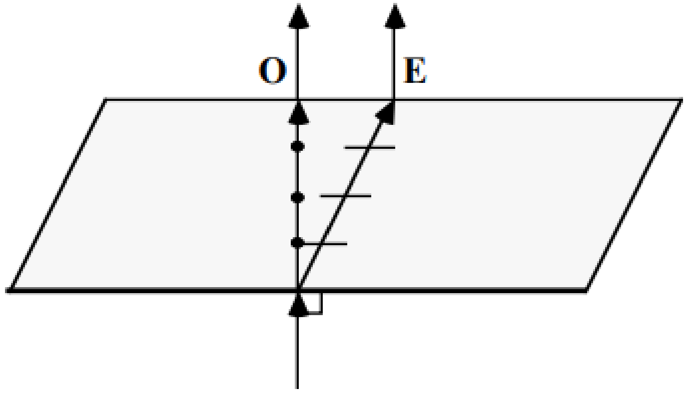
Fresnel-Arago Interference Laws

Double refraction in calcite,
Iceland spar

Fresnel-Arago Interference Laws

The experiment of Fresnel and Arago



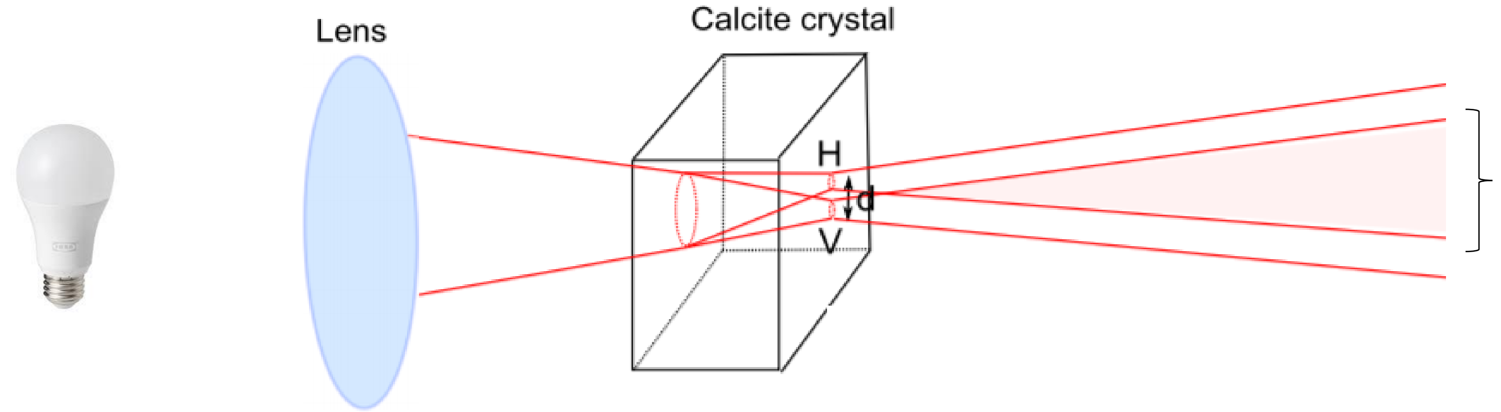


Only the ordinary wave satisfies the “usual” Snell law.

Plane containing the two beams and the optic axis of calcite

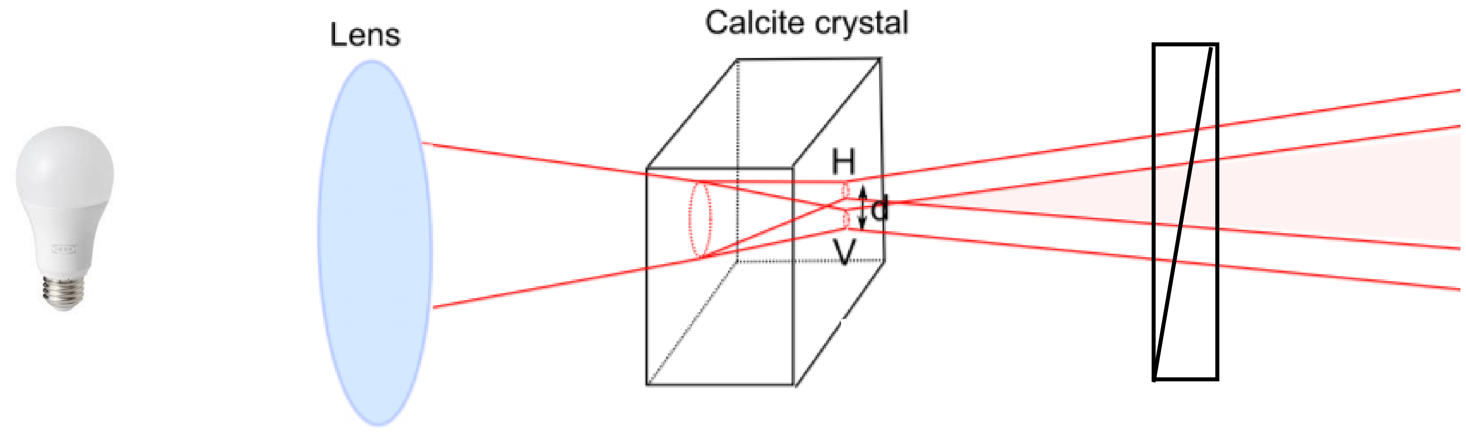
Fresnel-Arago Interference Laws

The experiment of Fresnel and Arago



Beam superposition but no interference

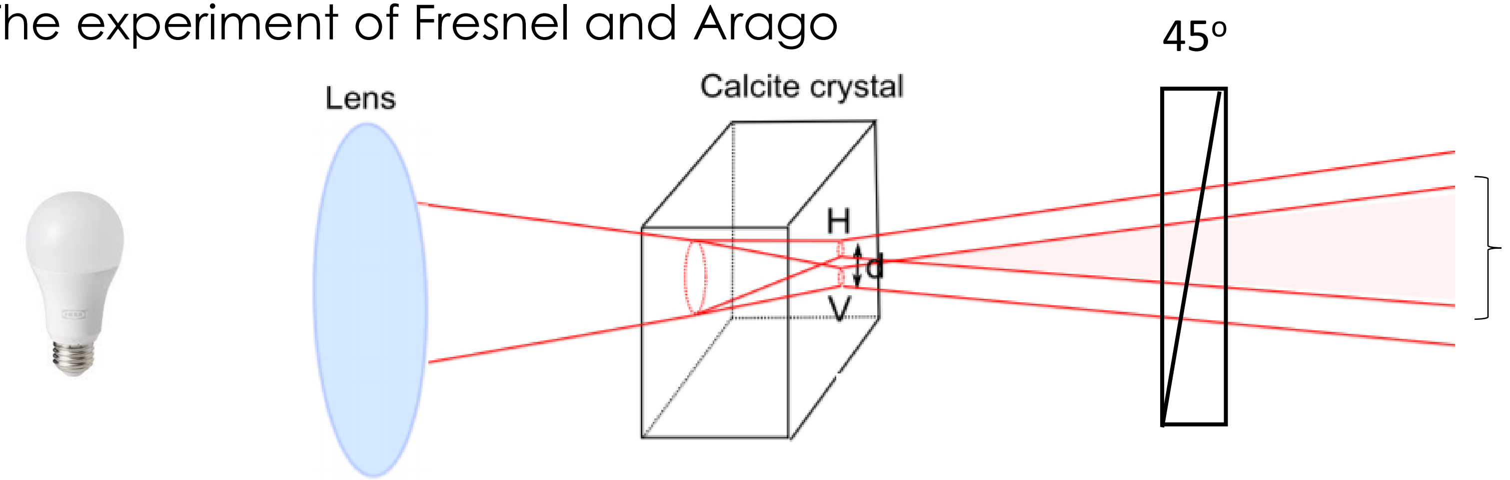
Polarizer at 45°



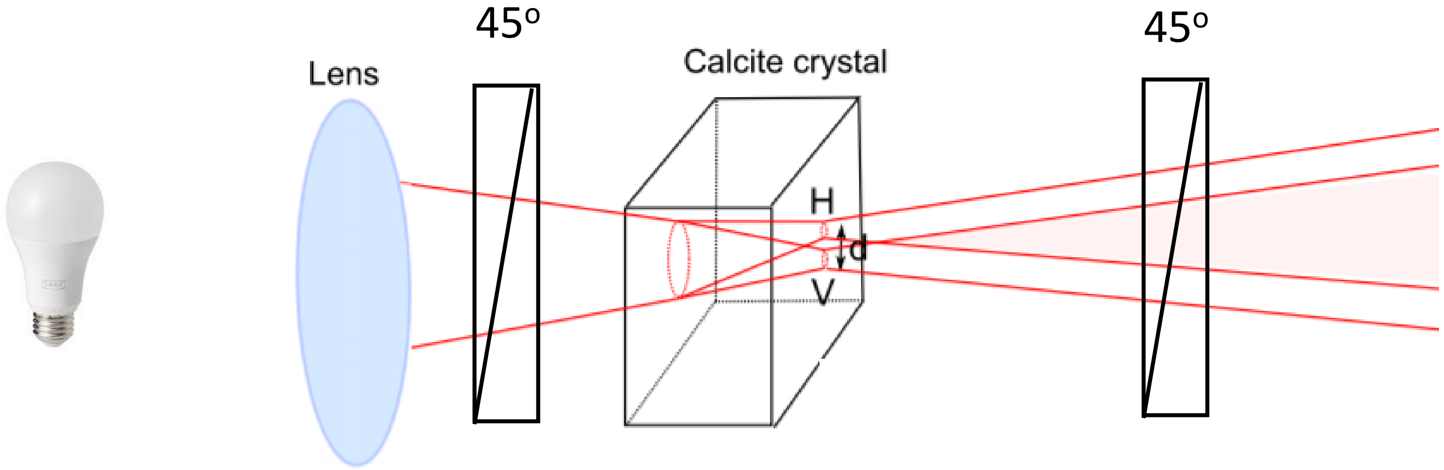
Interference?

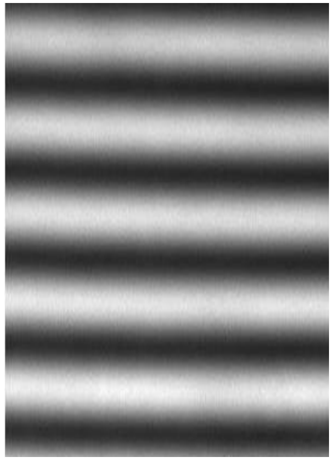
Fresnel-Arago Interference Laws

The experiment of Fresnel and Arago



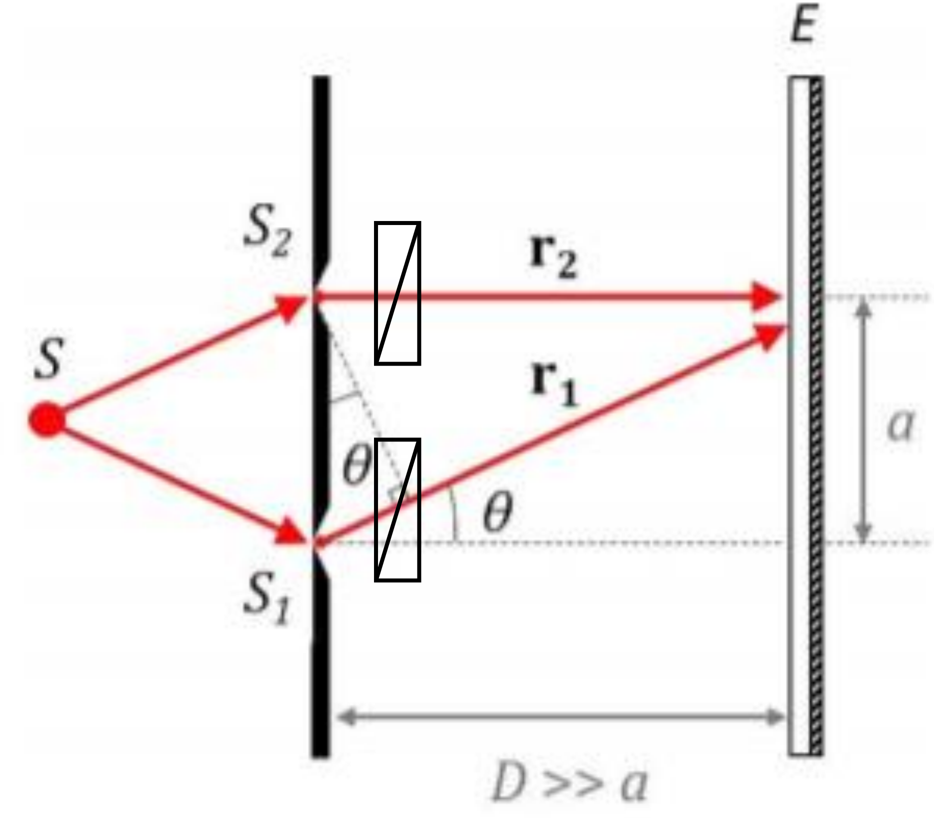
Beam superposition but no interference for natural (incoherent) light





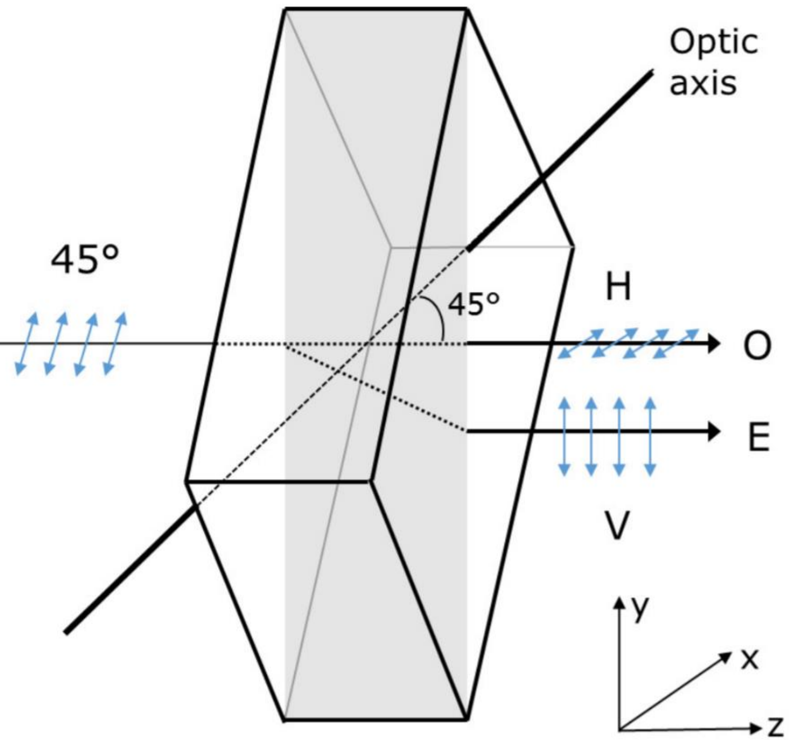
Fresnel-Arago Interference Laws

Young experiment with orthogonal polarizers



Analogy

Calcite experiment (split in H and V components)



Young's interference experiment is from 1799

History of polarization of light

1669 — **1808** — **1819** — **1956** — **now**

Rasmus Bartholin
Observed double refraction (but not explained in terms of polarization)
Iceland Spar (calcite)

Polarization of light by reflection
Malus Law


Arago and Fresnel
postulate the laws of interference and polarization

S. Pancharatnam discovered a geometric phase for polarized beams passing through crystals

3 Pancharatnam theory of interference

Pancharatnam theory of interference

Wesbite where you can find all publications of S. Pancharatnam while he was in India:

http://www.rri.res.in/htmls/library/raman_era/Publications/ps@rri.html

Shivaramakrishnan Pancharatnam (1934–1969)

- He died with only 35 years

Publications of Prof. S. Pancharatnam from RRI

1. On the Pleochroism of Amethyst Quartz and its Absorption Spectra
   Pancharatnam, S.
   Proceedings of the Indian Academy of Sciences A, 40, 196-210, 1954
2. Achromatic combinations of birefringent plates. Part 1 : An achromatic circular polarizer
   Pancharatnam, S.
   Proceedings of the Indian Academy of Sciences A, 41, 130-136, 1955
3. Achromatic combinations of birefringent plates. Part 2 : An achromatic quarter-wave plate
   Pancharatnam, S.
   Proceedings of the Indian Academy of Sciences A, 41, 137-144, 1955
4. The propagation of light in absorbing biaxial crystals - 1: Theoretical
   Pancharatnam, S.
   Proceedings of the Indian Academy of Sciences A, 42, 86-109, 1955
5. The propagation of light in absorbing biaxial crystals - 2: Experimental
   Pancharatnam, S.
   Proceedings of the Indian Academy of Sciences A, 42, 235, 1955
6. On the phenomenological theory of light propagation in optically active crystals
   Pancharatnam, S.
   Proceedings of the Indian Academy of Sciences A, 43, 247, 1956
7. Generalized theory of interference and its applications. Part 1: Coherent pencils
   Pancharatnam, S.
   Proceedings of the Indian Academy of Sciences A, 1956, 44, 247
8. Generalized theory of interference and its applications. Part 2: Partially coherent pencils
   Pancharatnam, S.
   Proceedings of the Indian Academy of Sciences A, 44, 398, 1956
9. Generalized theory of interference and its applications. Part 3: Interference figures in transparent crystals
   Pancharatnam, S.
   Proceedings of the Indian Academy of Sciences A, 45, 402, 1957
10. Generalizedtheory of interference and its applications. Part 4: Interference figures in absorbing biaxial crystals
    Pancharatnam, S.
    Proceedings of the Indian Academy of Sciences A, 46, 1, 1957
11. Light propagation in absorbing crystals possessing optical activity
    Pancharatnam, S.
    Proceedings of the Indian Academy of Sciences A, 46,280, 1957
12. The optic interference figures of amethystine quartz. Part 1
    Pancharatnam, S.
    Proceedings of the Indian Academy of Sciences A, 47, 201, 1958
13. The optic interference figures of amethystine quartz. Part 2
    Pancharatnam, S.
    Proceedings of the Indian Academy of Sciences A, 47, 210, 1958
14. Light propagation in absorbing crystals possessing optical activity - Electromagnetic theory
    Pancharatnam, S.
    Proceedings of the Indian Academy of Sciences A, 48, 227, 1958
15. The optics of mirages
    Raman, C.V.; Pancharatnam, S.
    Proceedings of the Indian Academy of Sciences A, 49, 251, 1959

Pancharatnam theory of interference

Reprinted from "The Proceedings of the Indian Academy of Sciences",
Vol. XLIV, No. 5, Sec. A, 1956

GENERALIZED THEORY OF INTERFERENCE, AND ITS APPLICATIONS

Part I. Coherent Pencils

BY S. PANCHARATNAM
(*Memoir No.* 88 *of the Raman Research Institute, Bangalore*-6)

Received October 30, 1956
(Communicated by Sir C. V. Raman)

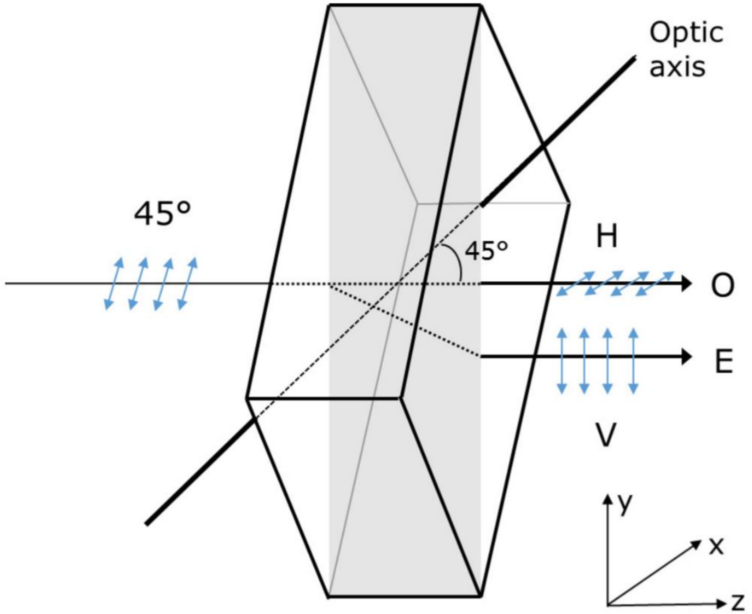
§ 1. INTRODUCTION

THE investigations of which the results are presented in this paper arose during the study of certain specific problems in crystal optics. As investigators in this field are well aware, the simplest procedures for studying the optical properties of anisotropic media (*e.g.*, examination under the polarising microscope) generally involve the use and study of polarised light. The

Pancharatnam theory of interference

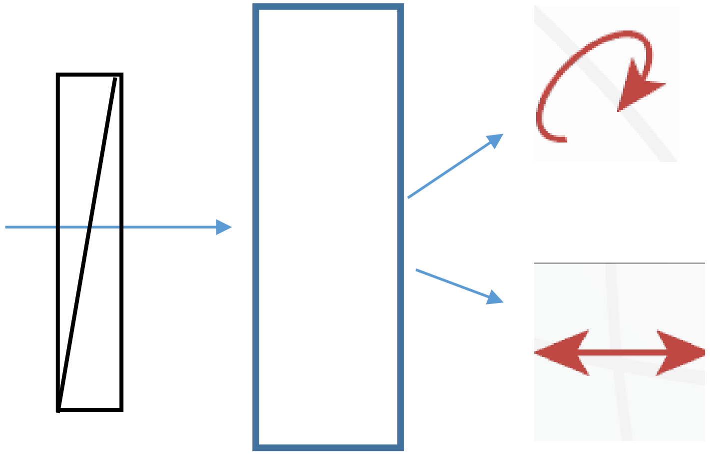
Pancharatman essentially studied the same type of problem that Fresnel and Arago had studied more than 100 years earlier but instead of using calcite, he used a more complex **biaxial absorbing crystal**

Fresnel-Arago



No
interference!

S. Pancharatnam



The output states are not ortogonal. **Some intereference is seen**

Absorbing biaxial crystal

Pancharatnam theory of interference

Pancharatman made two essential findings:

1. How does one define a phase difference between two light waves which are in different polarization states?

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beam is subjected to a particular path retardation relative to the second, then δ as defined above decreases by the corresponding phase angle; in the second place we note that as long as no path retardation is introduced between the two beams, any alteration of the intensities of the two beams will not change the value of δ as defined above. Hence we will be guilty of no internal inconsistency if we make the following statement by way of a definition: *the phase advance of one polarised beam over another* (not necessarily in the same state of polarisation) *is the amount by which its phase must be retarded relative to the second, in order that the intensity resulting from their mutual interference may be a maximum.*

This is sometimes called the **Pancharatnam Connexion**

Pancharatnam concluded that the phase difference between two beams is the phase change which when applied to one of them maximizes the intensity of their superposition

Pancharatnam theory of interference

Pancharatman made two essential findings:

1. How does one define a phase difference between two light waves which are in different polarization states?

Pancharatnam result can be expressed with the complex scalar product as:

Phase difference between $|A\rangle$ and $|B\rangle = \arg\langle A|B\rangle$

Example 1: Consider the Jones vectors

$$|E_A\rangle = \begin{pmatrix} \sqrt{3} \\ i \end{pmatrix}$$

$$\langle E_A|E_B\rangle = \begin{pmatrix} \sqrt{3} & -i \end{pmatrix}\begin{pmatrix} \sqrt{3}-i \\ \sqrt{3}+i \end{pmatrix} = 4 - 2\sqrt{3}i$$

They are not in phase

$$|E_B\rangle = \begin{pmatrix} \sqrt{3}-i \\ \sqrt{3}+i \end{pmatrix}$$

Pancharatnam theory of interference

Pancharatman made two essential findings:

1. How does one define a phase difference between two light waves which are in different polarization states?

Pancharatnam result can be expressed with the complex scalar product as:

Phase difference between $|A\rangle$ and $|B\rangle = \arg\langle A|B\rangle$

Example 2: Consider the Jones vectors

$$|E_A\rangle = \begin{pmatrix} \sqrt{3} \\ i \end{pmatrix}$$

$$|E_B\rangle = (2+\sqrt{3}i)\begin{pmatrix} \sqrt{3}-i \\ \sqrt{3}+i \end{pmatrix}$$

$$\langle E_A|E_B\rangle = (2+\sqrt{3}i)\begin{pmatrix} \sqrt{3} & -i \end{pmatrix}\begin{pmatrix} \sqrt{3}-i \\ \sqrt{3}+i \end{pmatrix} = 14$$

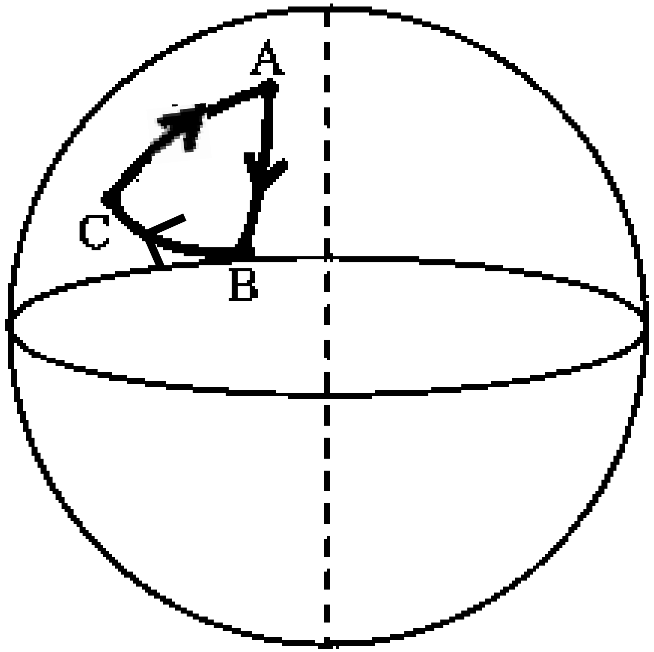
Real result
They are in phase!

Pancharatnam theory of interference

Pancharatman made two essential findings:

2. If a beam of light is taken along a closed circuit on the Poincaré Sphere, the beam acquires an extra phase equal to half the solid angle subtended by the closed circuit.

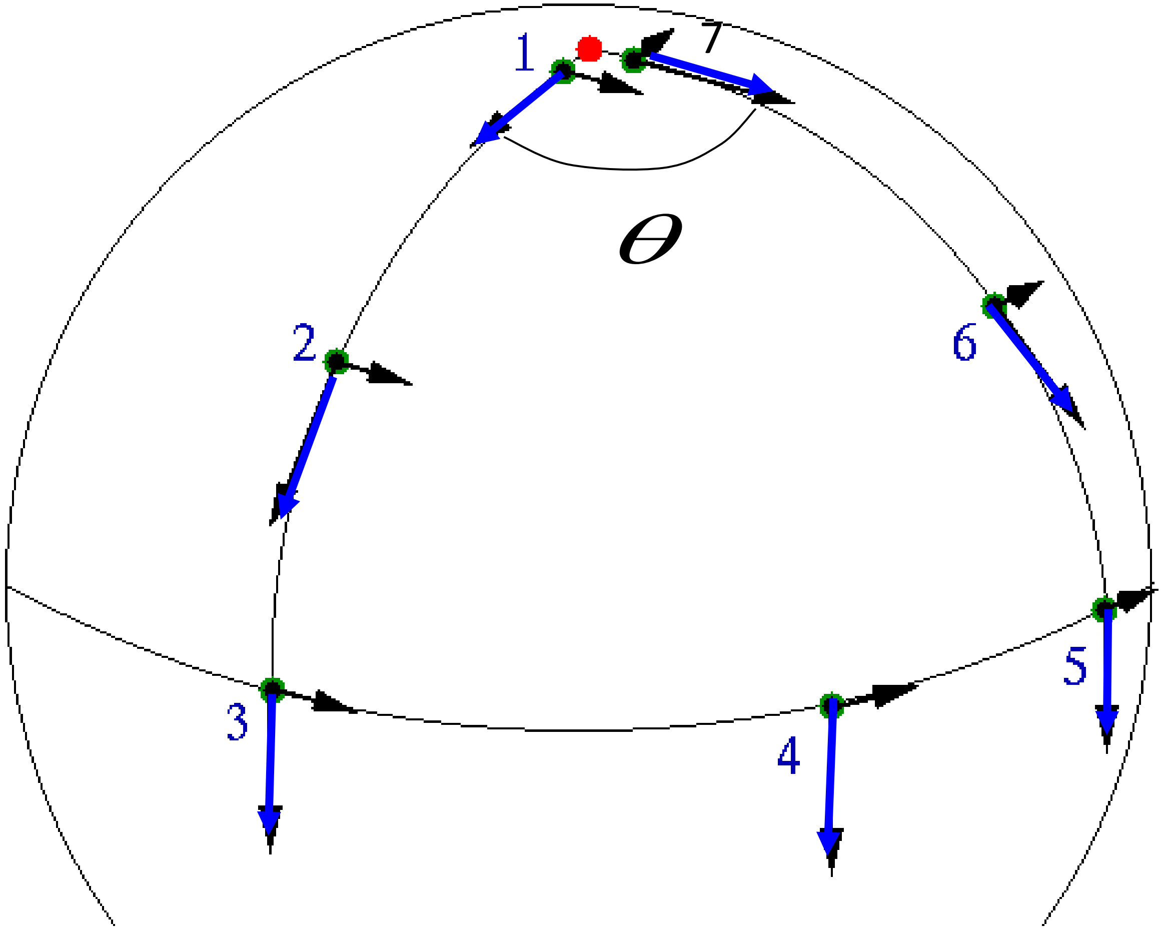
(in fact this statement is more general than what Pancharatnam wrote)



$$|A\rangle \rightarrow |B\rangle \rightarrow |C\rangle \rightarrow |A'\rangle$$

$$\langle A|A'\rangle = \exp\left(-\frac{1}{2} i\Omega_{ABC}\right)$$

Pancharatnam theory of interference



B. Goss Levi, Phys. Today 46, 17 (1993).

Lines which are **straight** on a **sphere** (or other surfaces) are often called **geodesics**.

Parallel transport: at each step, keep the vector as aligned to the previous one as possible.

Note that

- 1 is in phase with 3
- 3 is in phase with 5
- 1 is not in phase with 5/7

The blue vector is rotated by an angle which is equal to the solid angle subtended at the center enclosed by the loop: geometry of the space.

Pancharatnam theory of interference

The “rediscovery” of Pancharatman work

S. Pancharatman works had little or no impact at the time they were published. They were “rediscoved” in 1986/1987 after M.V Berry had published in 1984 a phase change for slowly cycled quantum systems

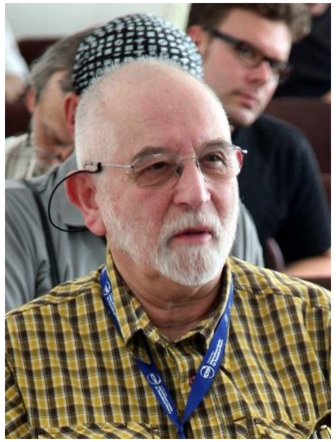
The adiabatic phase and Pancharatnam’s phase for polarized light

M. V. BERRY

H. H. Wills Physics Laboratory,
Tyndall Avenue, Bristol BS8 1TL, England

(*Received 17 August 1987*)

Abstract. In 1955 Pancharatnam showed that a cyclic change in the state of polarization of light is accompanied by a phase shift determined by the geometry of the cycle as represented on the Poincaré sphere. The phase owes its existence to the non-transitivity of Pancharatnam’s connection between different states of polarization. Using the algebra of spinors and 2×2 Hermitian matrices, the precise relation is established between Pancharatnam’s phase and the recently discovered phase change for slowly cycled quantum systems. The polarization phase is an optical analogue of the Aharonov–Bohm effect. For slow changes of polarization, the connection leading to the phase is derived from Maxwell’s equations for a twisted dielectric. Pancharatnam’s phase is contrasted with the phase change of circularly polarized light whose direction is cycled (e.g. when guided in a coiled optical fibre).



M. V. Berry from wikipedia

This is the reason why geometric pase in optics is often referred as **Pancharatnam-Berry (PB) phase**.

Pancharatnam theory of interference

All this seems complicate…

But the good part is that Jones calculus keeps always track of this **geometric phase**. We do not need to be good at geometry to take it into account.

Example:

Consider the following transformation:

Linear polarization → (QWP at 45º) → Circular polarization → (Rotating linear polarizer) → Linear polarization again

$$\mathbf{J}_{QWP+45} = \begin{pmatrix} \cos 45^{\circ} & -\sin 45^{\circ} \\ \sin 45^{\circ} & \cos 45^{\circ} \end{pmatrix} \begin{pmatrix} e^{-i\frac{\pi}{4}} & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix} \begin{pmatrix} \cos 45^{\circ} & \sin 45^{\circ} \\ -\sin 45^{\circ} & \cos 45^{\circ} \end{pmatrix} = \frac{\sqrt{2}}{2} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$$

$$\mathbf{J}_{\substack{rotated \\ polarizer}} = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} = \begin{pmatrix} \cos^2\varphi & \cos\varphi\sin\varphi \\ \cos\varphi\sin\varphi & \sin^2\varphi \end{pmatrix}$$

Pancharatnam theory of interference

Linear polarization → (QWP at 45°) → RH Circular polarization → (Rotating linear polarizer) → Linear polarization at an angle

$$\mathbf{J}_{\substack{rotated\\polarizer}}\mathbf{J}_{QWP+45}\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}\cos^2\varphi & \cos\varphi\sin\varphi\\ \cos\varphi\sin\varphi & \sin^2\varphi\end{pmatrix}\begin{pmatrix}1 & -i\\ -i & 1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}=e^{-i\varphi}\begin{pmatrix}\cos\varphi\\ \sin\varphi\end{pmatrix}$$

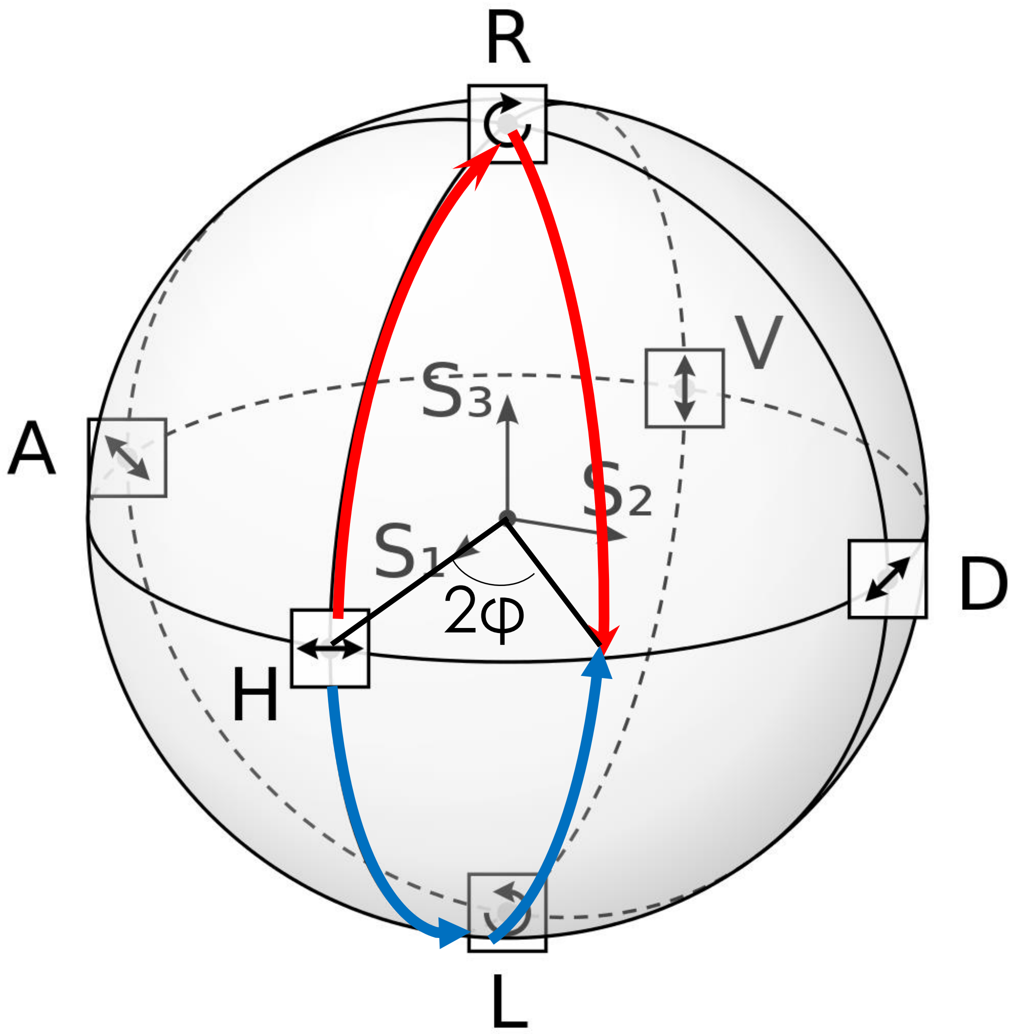
Linear polarization → (QWP at -45°) → LH Circular polarization → (Rotating linear polarizer) → Linear polarization at an angle

$$\mathbf{J}_{\substack{rotated\\polarizer}}\mathbf{J}_{QWP-45}\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}\cos^2\varphi & \cos\varphi\sin\varphi\\ \cos\varphi\sin\varphi & \sin^2\varphi\end{pmatrix}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}=e^{i\varphi}\begin{pmatrix}\cos\varphi\\ \sin\varphi\end{pmatrix}$$

Pancharatnam theory of interference

H Linear polarization → QWP at 45° → RH Circular polarization → Rotating linear polarizer → Linear polarization at an angle

H Linear polarization → QWP at -45° → LH Circular polarization → Rotating linear polarizer → Linear polarization at an angle

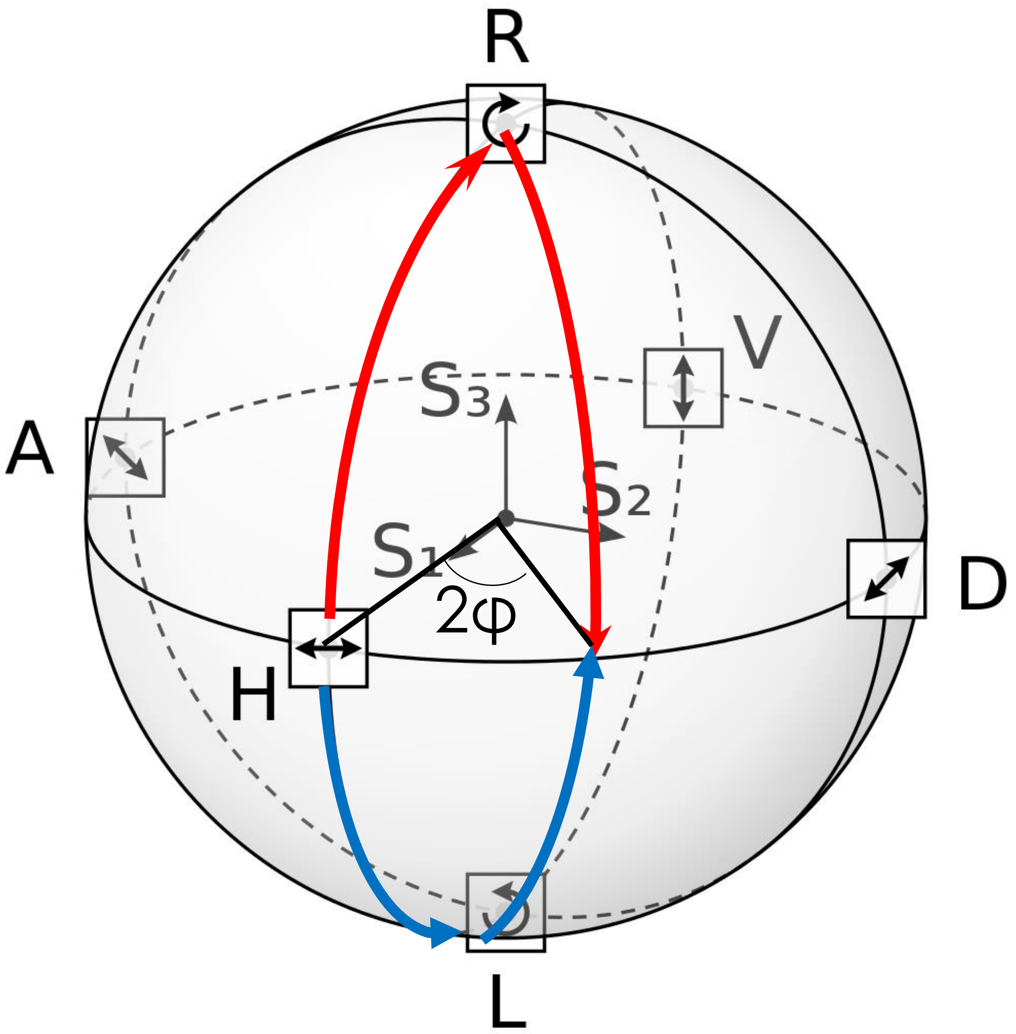


Note that the azimuthal angle on the Poincaré sphere is double the orientation angle of the polarizer

Pancharatnam theory of interference

The **solid angle** enclosed by these paths is $\Omega = 4\varphi$

$$\text{Geometric Phase} = \frac{1}{2}\Omega = 2\varphi$$



The same is verified using *Pancharatnam connection*:

$$|E_A\rangle = e^{-i\varphi}\begin{pmatrix}\cos\varphi \\ \sin\varphi\end{pmatrix}$$

$$\langle E_A | E_B\rangle = e^{i\varphi}e^{i\varphi}\left(\cos\varphi \quad \sin\varphi\right)\begin{pmatrix}\cos\varphi \\ \sin\varphi\end{pmatrix} = e^{i2\varphi}$$

$$|E_B\rangle = e^{i\varphi}\begin{pmatrix}\cos\varphi \\ \sin\varphi\end{pmatrix}$$

Note that the azimuthal angle on the Poincaré sphere is double the orientation angle of the polarizer

Pancharatnam theory of interference

- A beautiful comparison

The Pancharatnam phase is just one example of a general mathematical phenomenon which is mirrored in different physical contexts. Stated in common language, it seems that one does one's best to see that something (the phase of a polarised beam in this case) does not change, but when one comes back to the starting point after a circuitous journey, it has changed. *One example is from a story by Jules Verne. The eccentric Englishman, Phileas Fogg travels around the world, keeping careful track of days since he has staked his entire fortune on a bet to finish the journey in eighty days. Since he arrives a few minutes too late after seeing eighty sunsets, he assumes he has lost the bet. But his resourceful companion, Passepartout, (naturally a Frenchman in Verne's novel!) finds out that in London, only seventy nine days have passed, and Fogg is able to win his wager after all, and marry his Indian bride as well*

R. Nityananda, "The interference of polarized light", Resonance April 2013, page 321

4 Metasurfaces and geometric phase optical elements

Metasurfaces and geometric phase optical elements

Consider the Jones matrix of half wave (HWP) retarder

$$\mathbf{J} = \begin{pmatrix} e^{-i\delta/2} & 0 \\ 0 & e^{i\delta/2} \end{pmatrix} \xrightarrow{\delta=\pi} \begin{pmatrix} e^{-i\pi/2} & 0 \\ 0 & e^{i\pi/2} \end{pmatrix} = e^{-i\pi/2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Now we rotate it by an angle ϕ

$$\mathbf{J}' = e^{-i\pi/2} \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} = e^{-i\pi/2} \begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{pmatrix}$$

Metasurfaces and geometric phase optical elements

Let's study what happens when this Jones matrix is projected on a Stokes vector of circular polarization

$$\mathbf{J}'\mathbf{E}_{\pm} = e^{-\pi i/2} \begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{pmatrix} \begin{pmatrix} 1 \\ \pm i \end{pmatrix} = e^{-\pi i/2} e^{\pm 2\phi i} \begin{pmatrix} 1 \\ \mp i \end{pmatrix}$$

A phase that depends on the orientation angle appears

This can have a technological relevance if $\phi(x, y)$

A wave will be subjected to a transversely inhomogeneous polarization transformation that will produce a waveform reshaping depending on the polarization of light, opening the possibility for the so-called Pancharatnam-Berry phase optical elements

Metasurfaces and geometric phase optical elements

For comparison purposes we can do the same calculus using the Stokes-Mueller formalism

- A half wave plate

$$\mathbf{M}_{HWP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- A rotated half-wave plate

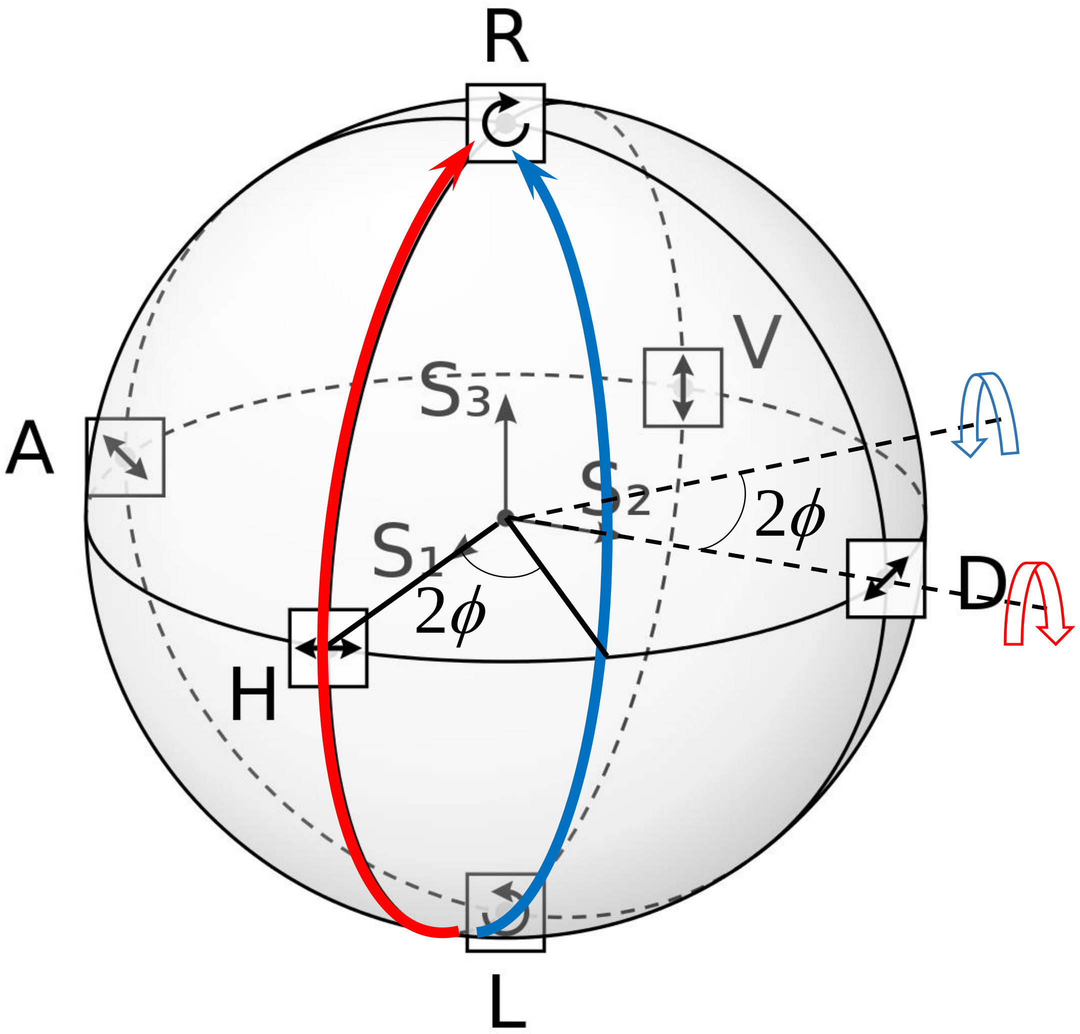
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\phi & -\sin 2\phi & 0 \\ 0 & \sin 2\phi & \cos 2\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\phi & \sin 2\phi & 0 \\ 0 & -\sin 2\phi & \cos 2\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 4\phi & \sin 4\phi & 0 \\ 0 & \sin 4\phi & -\cos 4\phi & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 4\phi & \sin 4\phi & 0 \\ 0 & \sin 4\phi & -\cos 4\phi & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ \pm 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \mp 1 \end{pmatrix}$$

As expected, the phase is not visible in this formalism

Metasurfaces and geometric phase optical elements

What is the effect of the rotated HWP on the Poincaré Sphere?



Example:

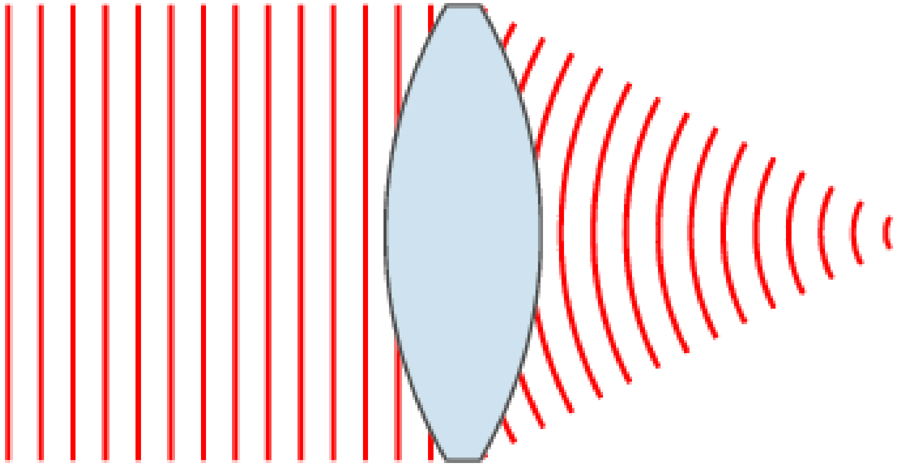
First we consider a HWP at 45° acting on a beam of left circular polarization

Second we consider a HWP at 45°+ϕ acting on a beam of left circular polarization

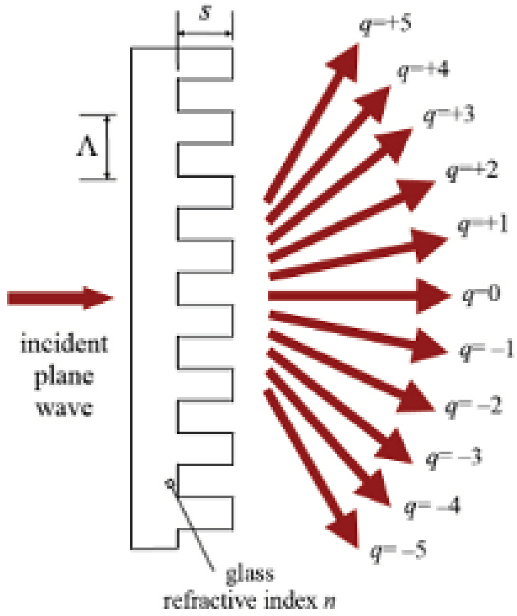
$$\text{Geometric Phase} = \frac{1}{2}\Omega = 2\phi$$

Metasurfaces and geometric phase optical elements

How does a regular lens or a grating work? They produce phase changes due to **thickness** changes



As the wave hits the lens, it is the centre of the wave that meets the glass first and so this part of the wave is slowed down first. The outer portions of the wave 'catch up' so increasing the curvature to form a converging beam

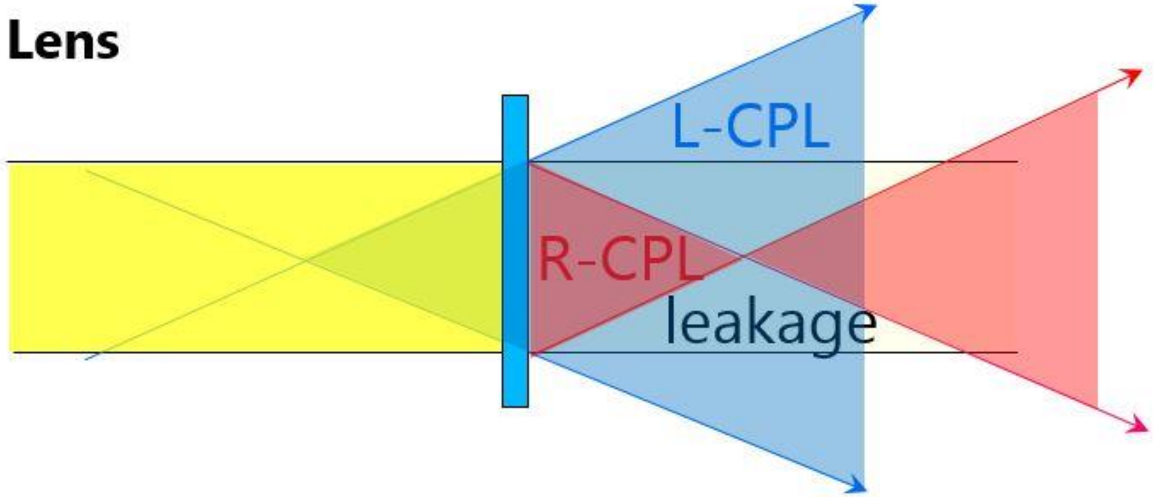


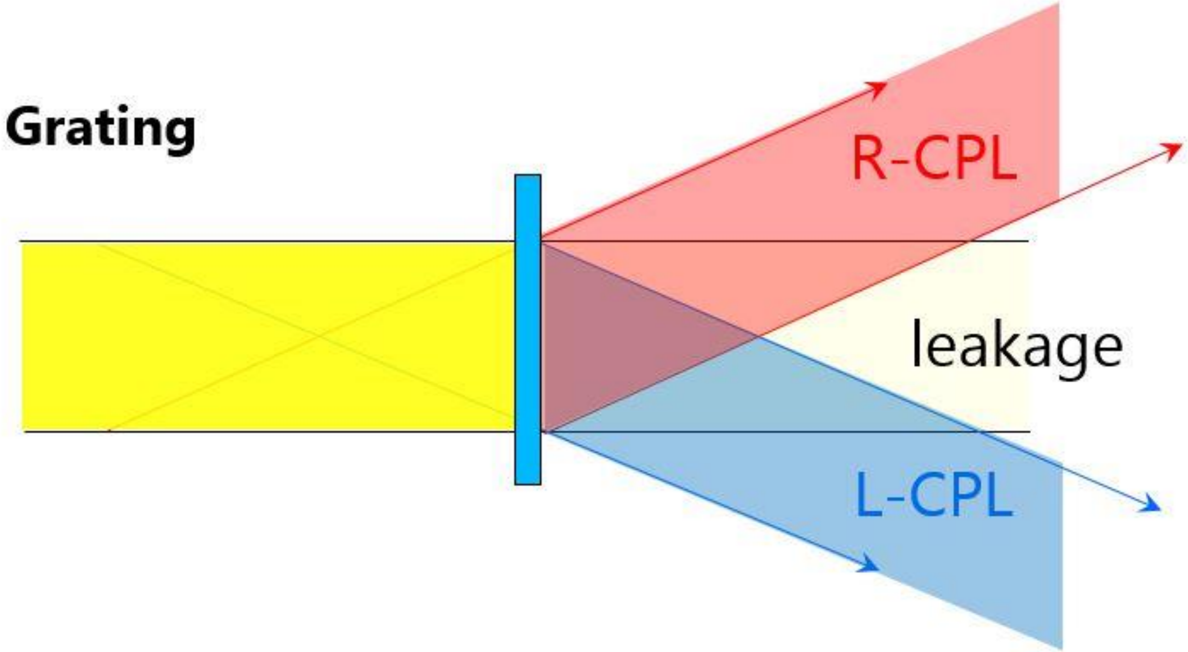
A transmission grating is a periodical structure with steps (binary here) that diffracts light

Metasurfaces and geometric phase optical elements

What is the effect of a GP optical element? A wavefront reshaping that depends on the orientation, not on the **thickness. A GP optical element can be flat!**

$$\mathbf{J}'\mathbf{E}_{\pm} = e^{-\pi i/2} \begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{pmatrix} \begin{pmatrix} 1 \\ \pm i \end{pmatrix} = e^{-\pi i/2} e^{\pm 2\phi i} \begin{pmatrix} 1 \\ \mp i \end{pmatrix}$$





Metasurfaces and geometric phase optical elements

It is possible to rewrite the Jones matrix of a rotated retarder as follows

$$\begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{pmatrix} = \frac{e^{-2\phi i}}{2}\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \frac{e^{2\phi i}}{2}\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

What happens if we send light with a polarization that is different from circular?

$$\begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos 2\phi \\ \sin 2\phi \end{pmatrix} = \underbrace{\frac{e^{-2\phi i}}{2}\begin{pmatrix} 1 \\ i \end{pmatrix} + \frac{e^{2\phi i}}{2}\begin{pmatrix} 1 \\ -i \end{pmatrix}}$$

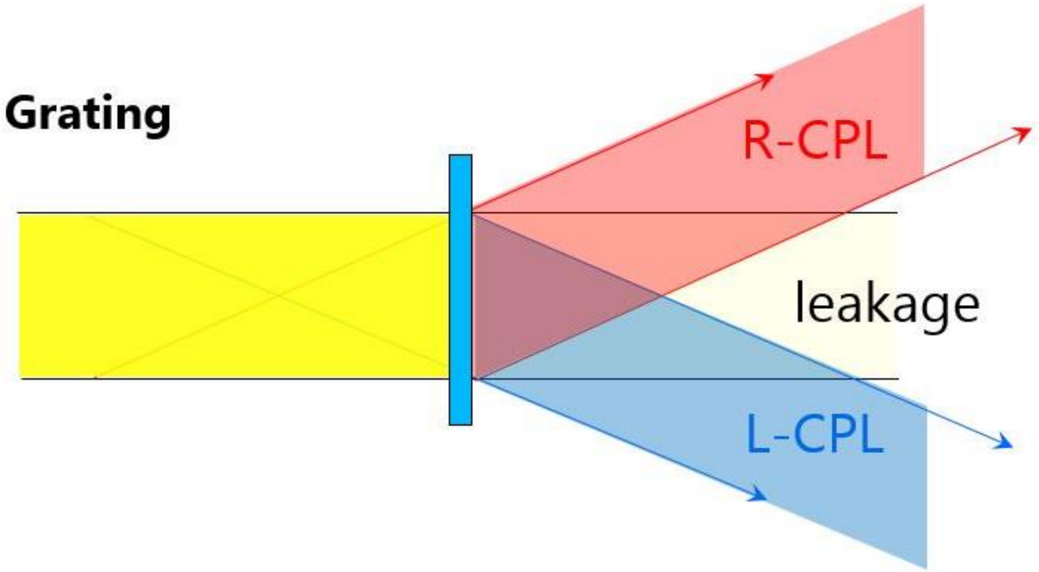
The opposite signs of the phases split into the two orthogonal circular components

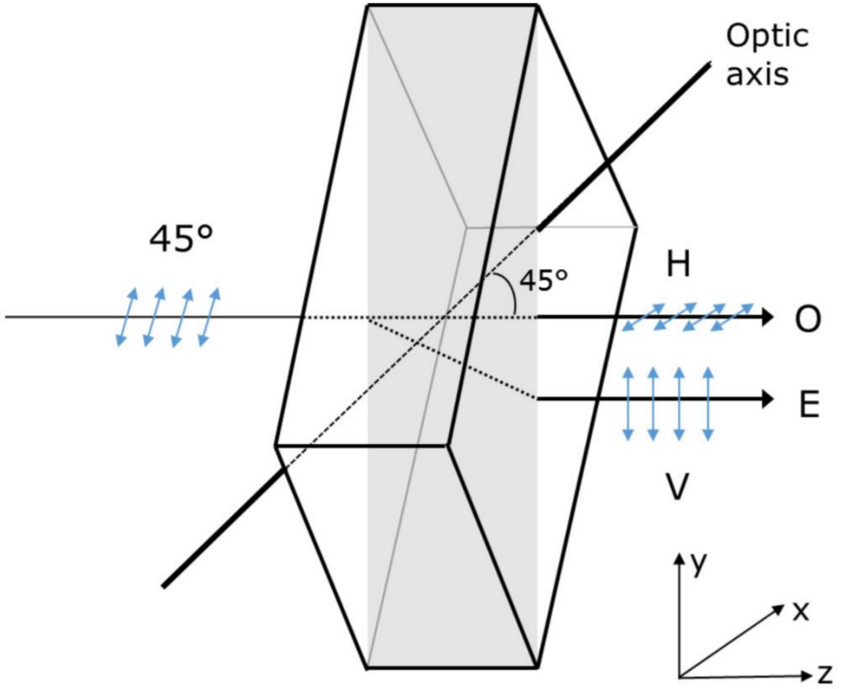
Metasurfaces and geometric phase optical elements

Photo by
Hana Bendada

Metasurfaces and geometric phase optical elements

This experiment is a good experimental illustration of something that theoretically presented in many Physics courses: any beam of light can be resolved into its **circular components** (in other words, one can use the circular basis to describe any polarization problem)





Metasurfaces and geometric phase optical elements

We discuss now **the existence of a leakage wave**. We consider a retardance different from π

$$\mathbf{J} = \begin{pmatrix} e^{-i\delta/2} & 0 \\ 0 & e^{i\delta/2} \end{pmatrix} \xrightarrow{\delta=\pi+\Delta} \begin{pmatrix} e^{-i(\pi+\Delta)/2} & 0 \\ 0 & e^{i(\pi+\Delta)/2} \end{pmatrix} = e^{-i\pi/2} \begin{pmatrix} e^{-i\Delta/2} & 0 \\ 0 & -e^{i\Delta/2} \end{pmatrix} =$$

$$= e^{-i(\pi+\Delta)/2} \left[\frac{1-e^{i\Delta}}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1+e^{i\Delta}}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right]$$

Now we rotate it by an angle ϕ

If $\Delta = 0$ this term vanishes

$$\mathbf{J}' = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \mathbf{J} \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} = e^{-i(\pi+\Delta)/2} \left[\frac{1-e^{i\Delta}}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1+e^{i\Delta}}{2} \begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{pmatrix} \right]$$

Metasurfaces and geometric phase optical elements

Leak term(independent from ϕ)

Wavefront reshaping

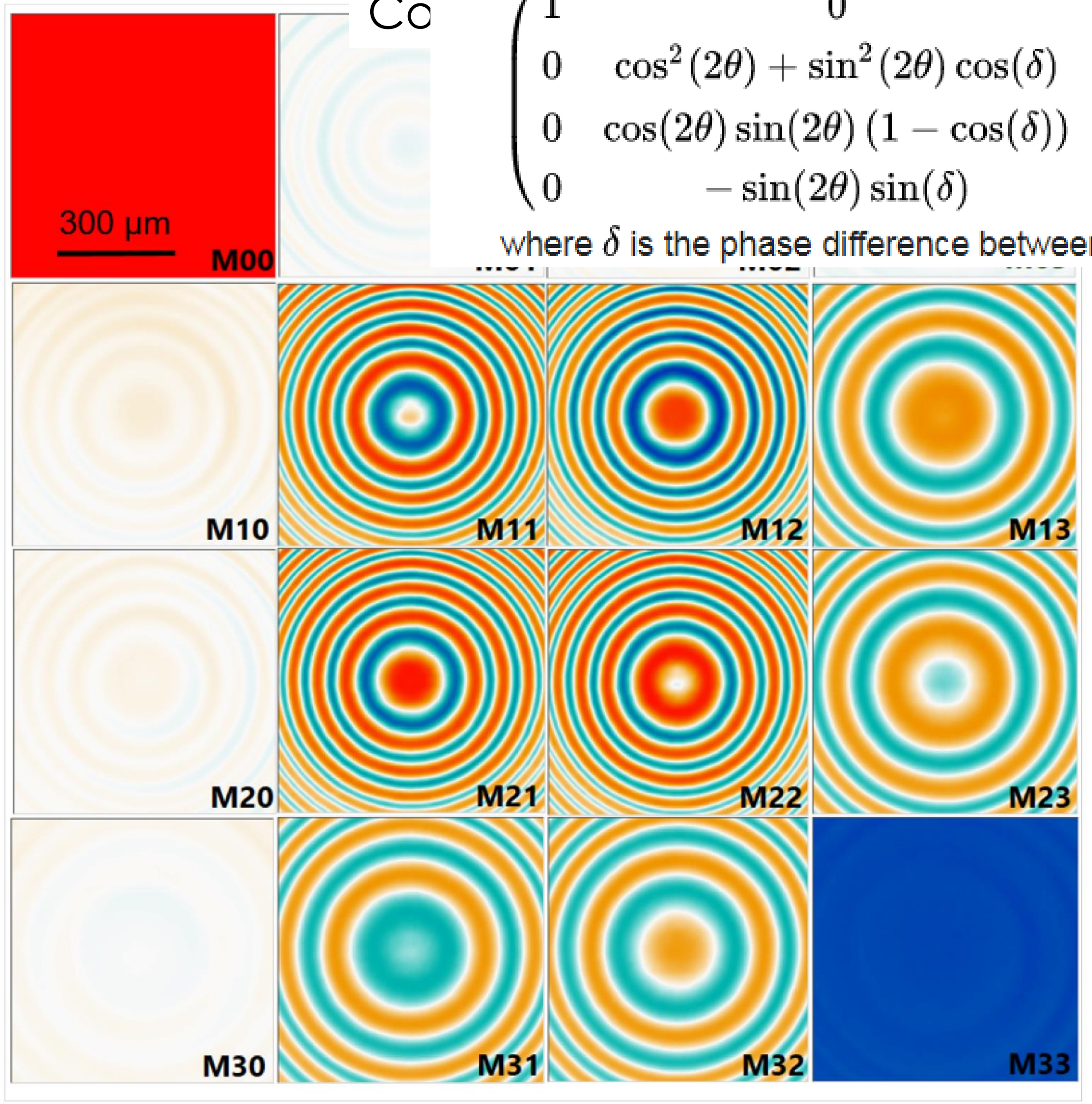
$$\mathbf{J}' = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \mathbf{J} \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} = e^{-i(\pi+\Delta)/2} \left[\frac{1-e^{i\Delta}}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1+e^{i\Delta}}{2} \begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{pmatrix} \right]$$

$$\begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{pmatrix} = \frac{e^{-2\phi i}}{2} \begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \frac{e^{2\phi i}}{2} \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\mathbf{E}_{in} \rightarrow A_{leak}\mathbf{E}_{leak} + A_{+}e^{2\phi i}\mathbf{E}_{+} + A_{-}e^{-2\phi i}\mathbf{E}_{-}$$

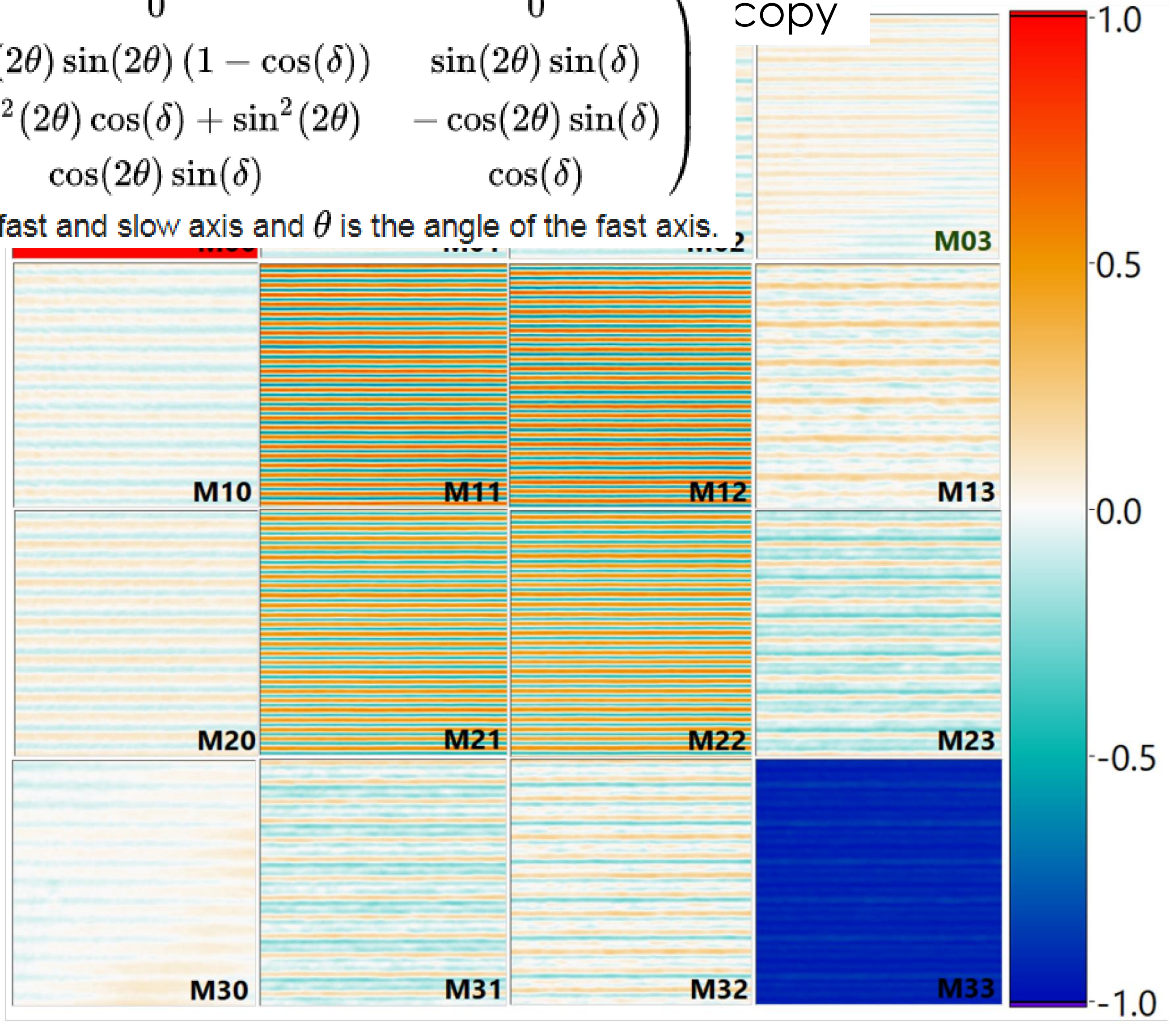
More info: Arteaga, O.; Bendada, H. Geometrical Phase Optical Components: Measuring Geometric Phase without Interferometry. *Crystals* **2020**, *10*, 880.

Metasurfaces and geometric phase optical elements

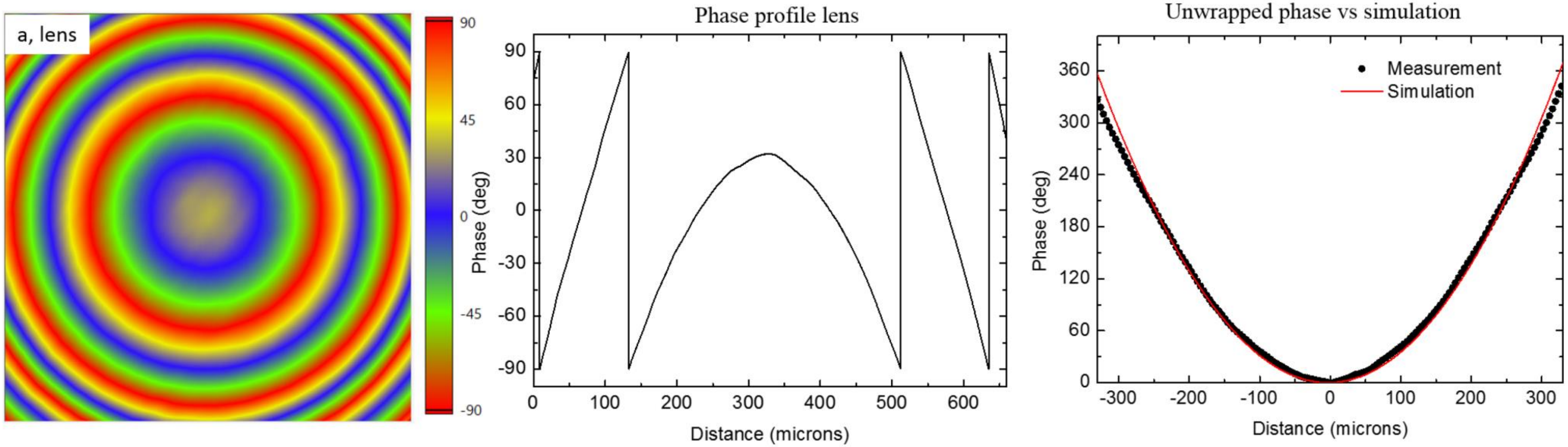


$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2(2\theta)+\sin^2(2\theta)\cos(\delta) & \cos(2\theta)\sin(2\theta)\,(1-\cos(\delta)) & \sin(2\theta)\sin(\delta) \\ 0 & \cos(2\theta)\sin(2\theta)\,(1-\cos(\delta)) & \cos^2(2\theta)\cos(\delta)+\sin^2(2\theta) & -\cos(2\theta)\sin(\delta) \\ 0 & -\sin(2\theta)\sin(\delta) & \cos(2\theta)\sin(\delta) & \cos(\delta) \end{pmatrix}$$

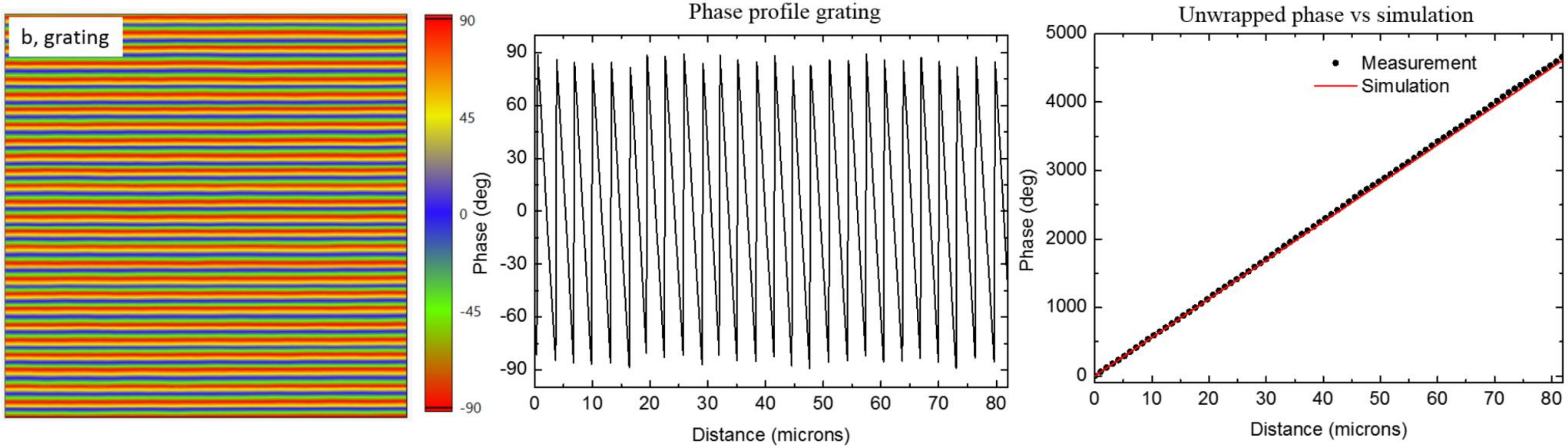
where δ is the phase difference between the fast and slow axis and θ is the angle of the fast axis.



Metasurfaces and geometric phase optical elements



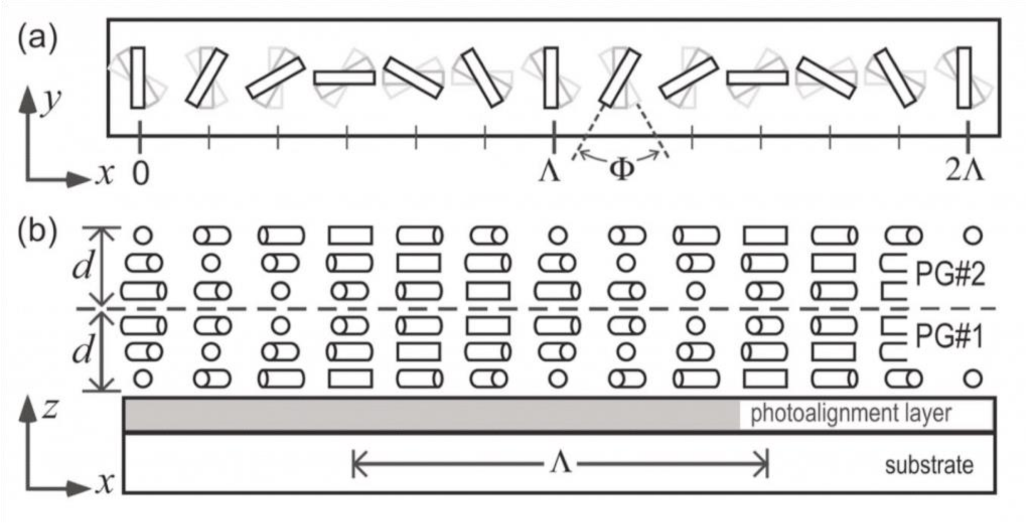
$$\phi_{lens}(r) = \frac{\pi}{\lambda}\left(\sqrt{f^2 + r^2} - f\right)$$

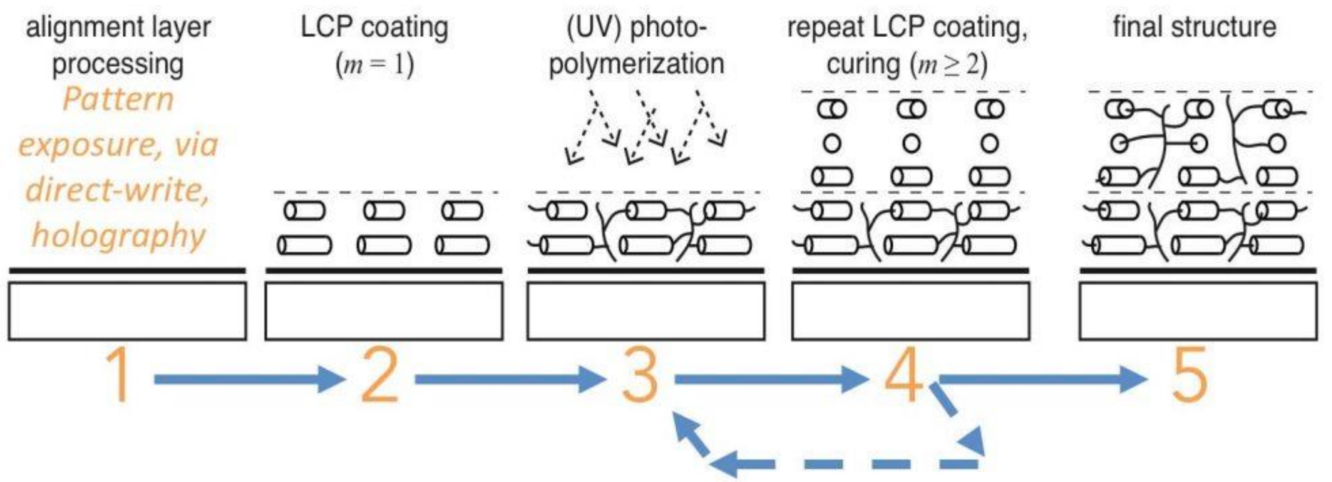


$$\phi_{grating}(x) = \frac{\pi x}{\Lambda}$$

Metasurfaces and geometric phase optical elements

The GP optical component is usually made from the polymerized liquid crystal (LC) where the orientation state of the LC director induces the phase variation



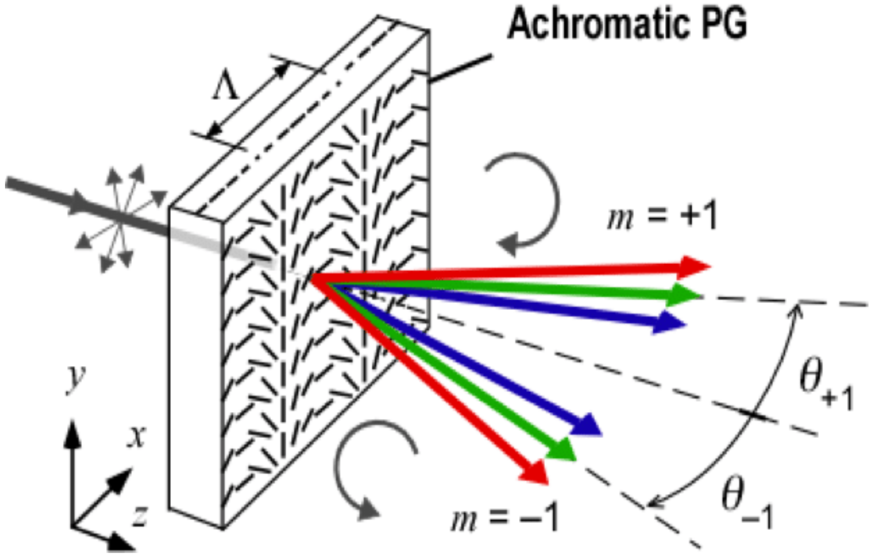


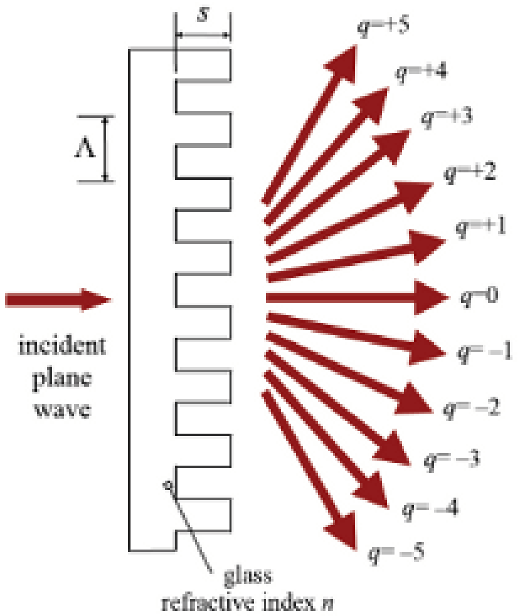
From: https://imagineoptix.com/

Multiple layers makes the half wave retardation more achromatic

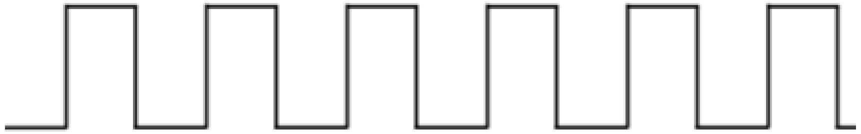
Metasurfaces and geometric phase optical elements

What about higher diffraction orders?





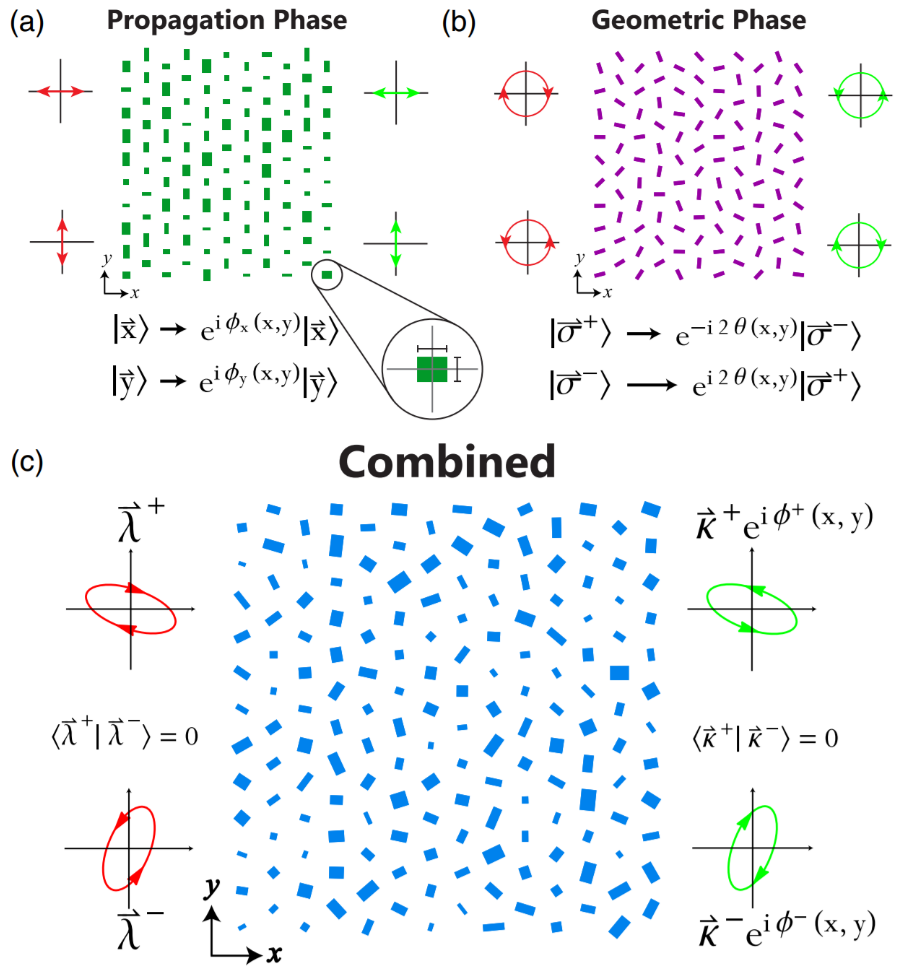
From: https://imagineoptix.com/

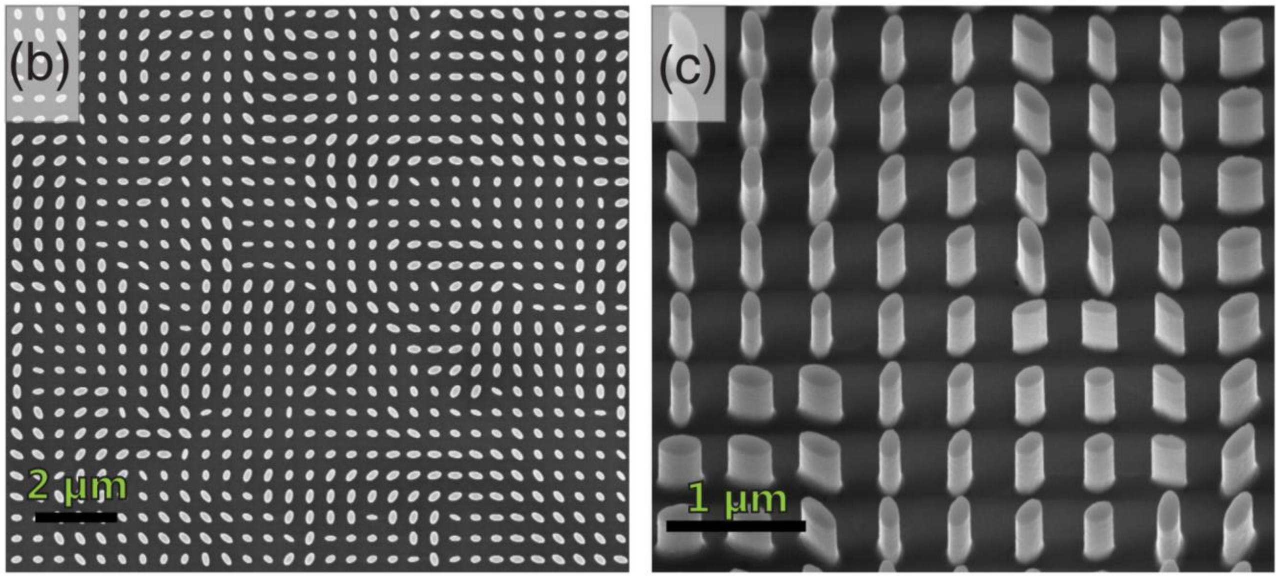


A sinusoid grating does not generate higher order diffraction orders

Metasurfaces and geometric phase optical elements

More sophisticated designs: Controlling not only the phase angle point by point, but also the amount of retardation. This adds an extra degree of freedom.





Metasurface with TiO_2 Pillars with shape anisotropy

J. P. BALTHASAR MUELLER, Noah A. Rubin, Robert C. Devlin, Benedikt Groever, and Federico Capasso. 2017. "Metasurface Polarization Optics: Independent Phase Control of Arbitrary Orthogonal States of Polarization." Physical Review Letters, 118, 11, Pp. 113901.

5. One more thing

History of polarization of light


1669 — 1808 — 1819 — 1822 — 1956 — now

Rasmus Bartholin
Observed double refraction (but not explained in terms of polarization)
Iceland Spar (calcite)

Polarization of light by reflection
Malus Law

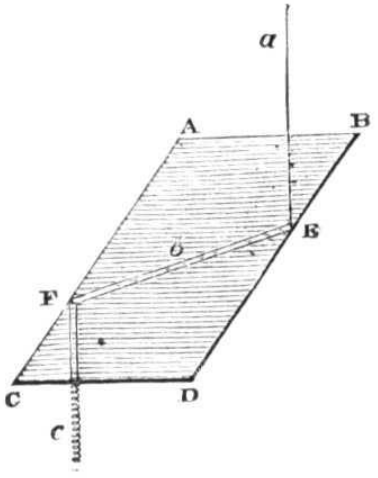
Agustin Fresnel coins the terms “circular polarization”, “elliptical polarization” and “ linear polarization”

S. Pancharatnam discovered a geometric phase for polarized beams passing through crystals

History of the circular polarization of light

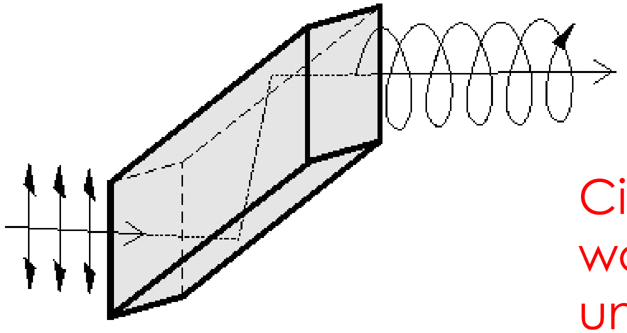
What was known in 1820

- Light as a transverse wave
- Double refraction ⟺ Linear polarization (the only known type of "polarization")



After two total internal reflections
light seemed "*depolarized*"

A. Fresnel, "Mémoire sur les modifications que la réflexion imprime à la lumière polarisée" ("Memoir on the modifications that reflection impresses on polarized light"), 10 November 1817,



Circular polarization was not yet understood!!

History of the circular polarization of light

N° XXVIII.

MÉMOIRE

SUR

LA DOUBLE RÉFRACTION

QUE LES RAYONS LUMINEUX

ÉPROUVENT EN TRAVERSANT LES AIGUILLES DE CRISTAL DE ROCHE

SUIVANT DES DIRECTIONS PARALLÈLES À L'AXE[a].

PRÉSENTÉ À L'ACADÉMIE DES SCIENCES LE 9 DÉCEMBRE 1822.

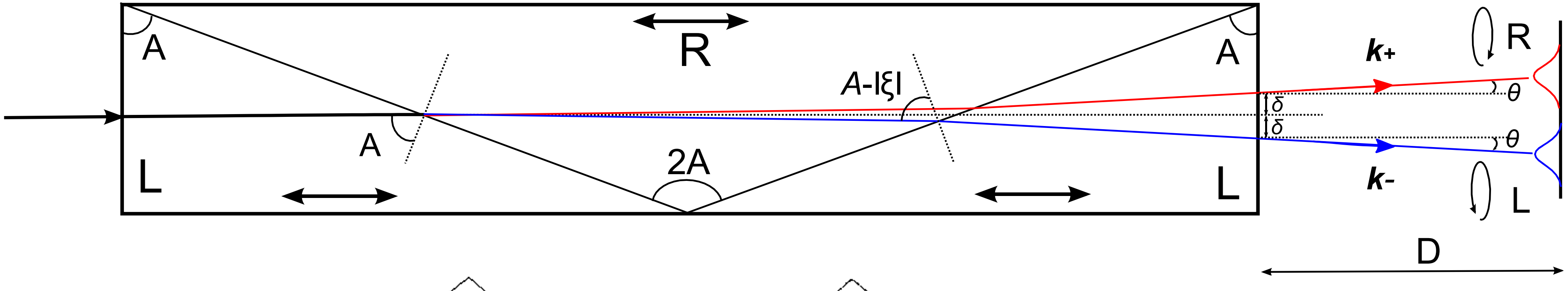
In the last part of the memoir

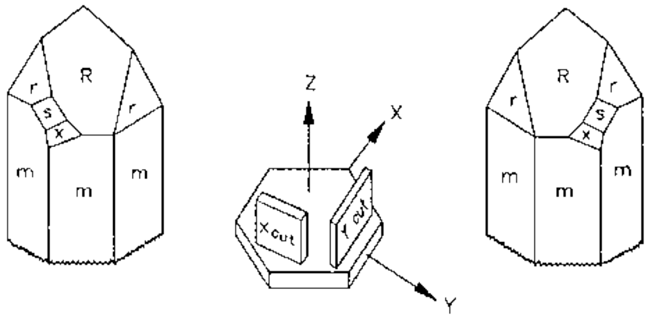
9. D'après la seule considération des faits, on pourrait donner le nom de *polarisation rectiligne* à celle qu'on avait observée depuis longtemps dans la double réfraction du spath calcaire, et que Malus a le premier remarquée dans la lumière réfléchie sur les corps transparents, et nommer *polarisation circulaire* la nouvelle modification dont je viens de décrire les propriétés caractéristiques : elle se divisera naturellement en *polarisation circulaire de gauche à droite, et polarisation circulaire de droite à gauche*. Ces dénominations, qui m'ont été suggérées par l'hypothèse que j'ai adoptée sur les vibrations lumineuses, indiquent la nature même de leurs mouvements dans les deux cas; mais, craignant d'abuser des moments de l'Académie, j'ai cru devoir me borner ici à justifier les noms nouveaux que je propose par la simple exposition des faits. Les développements théoriques trouveront naturellement leur place dans un supplément, que je joindrai à ce Mémoire[a].

10. Entre la polarisation rectiligne et la polarisation circulaire, il existe une foule de degrés intermédiaires de polarisations diverses, qui participent des caractères des deux autres, et auxquels on pourrait donner les noms de *polarisations elliptiques*, d'après les mêmes vues théoriques. On peut produire divers genres de polarisation, soit par une

History of the circular polarization of light

Fresnel's quartz triprism, the circular double refraction





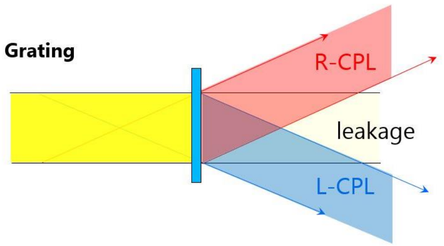
Left-hand Crystal

Right-hand Crystal

$$\theta \simeq 2\Delta \tan A$$

$$\Delta = |n_R - n_L|$$

A=76°



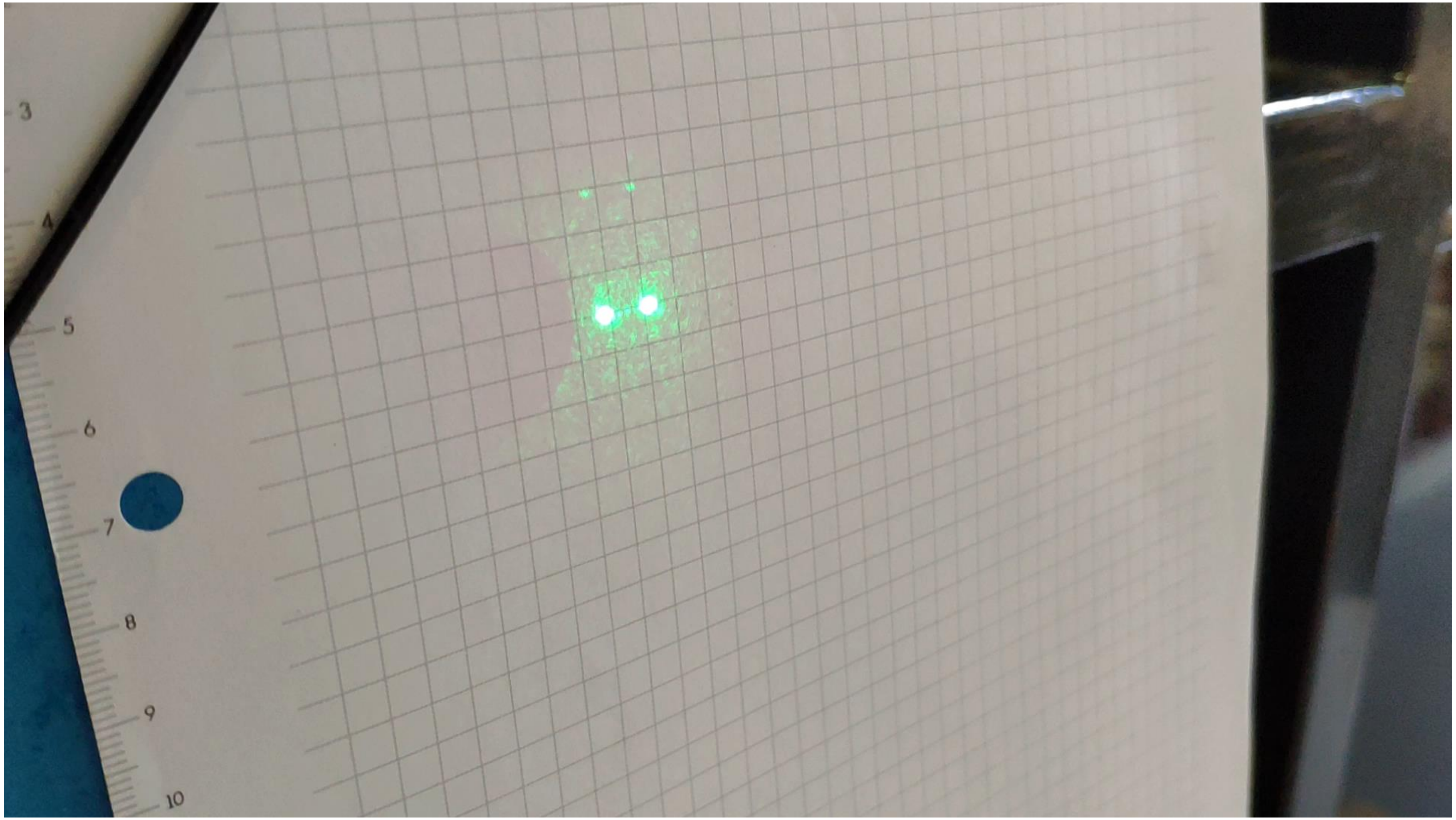
Note that the effect look similar to a geometric phase grating

History of the circular polarization of light

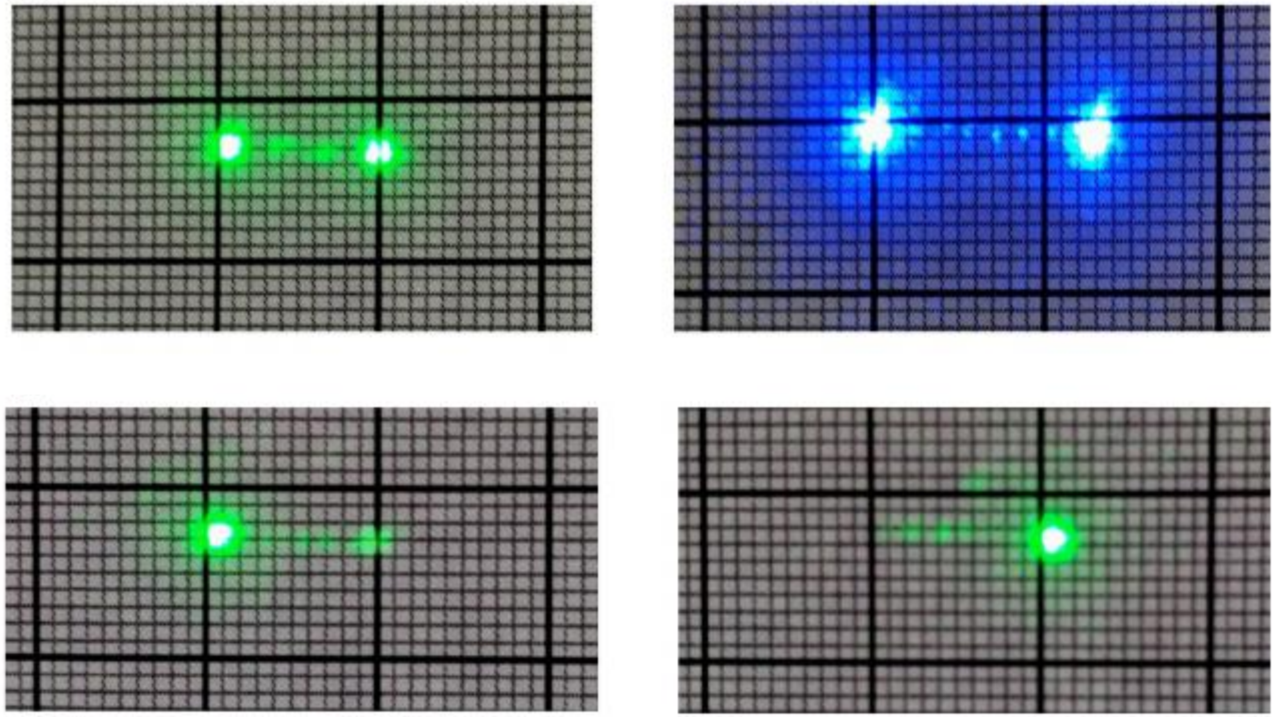
Quartz polyprism of the Museum of the École Polytechnique

History of the circular polarization of light

History of the circular polarization of light

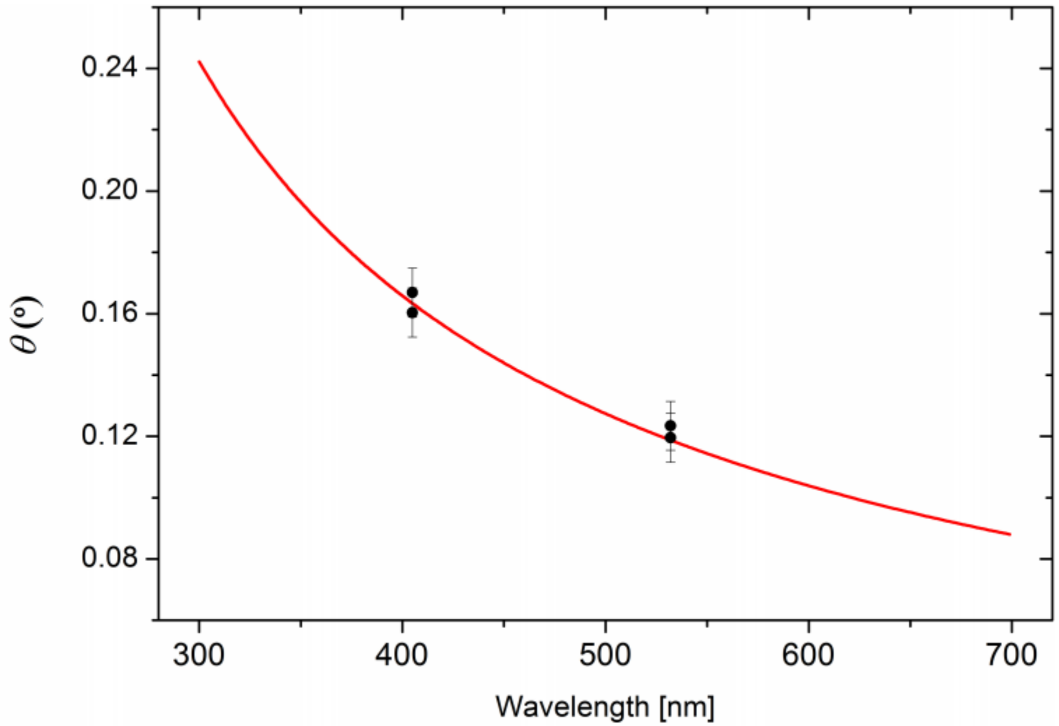


History of the circular polarization of light



O. Arteaga, E. Garcia-Caurel, and Razvigor Ossikovski, Opt. Express 27, 4758-4768 (2019)

Beam deflection vs wavelength

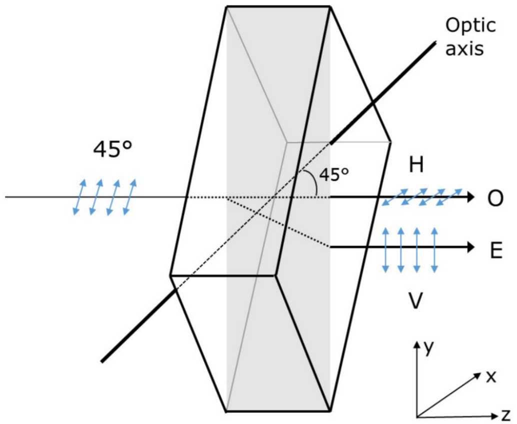


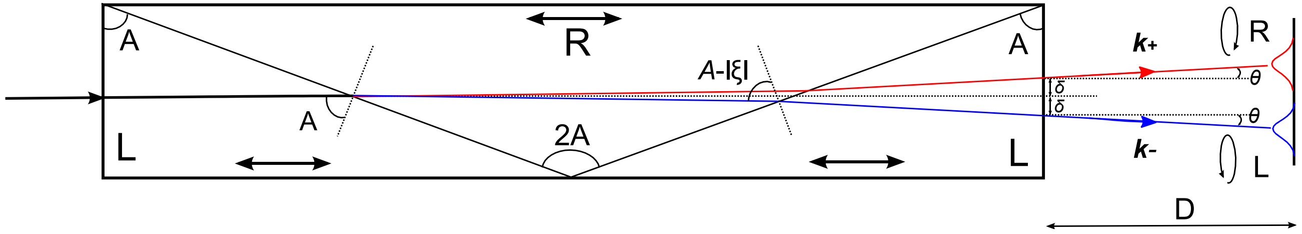
$$\theta \simeq N\Delta \tan A$$

$$\Delta = |n_R - n_L|$$

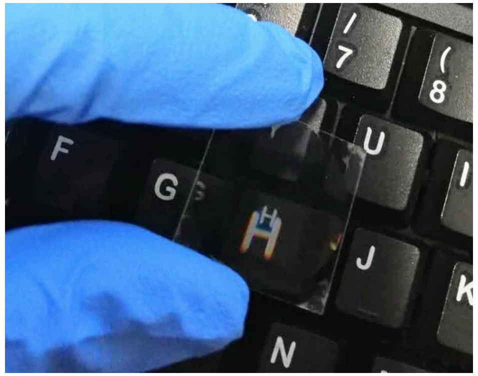
Polarized light can be split into components

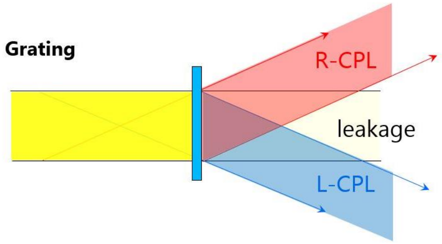
With anisotropic/optically active materials in which the index of refraction depends on the polarization





With metasurfaces in which the sign of the geometric phase depends on the polarization





Thank you!

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