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MEMORANDUM ON THE POLARIZATION-OPTICS OF THE PHOTOELECTRIC SHUTTER

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NDRC Project No. OEMsr-576
Hans Mueller, Official Investigator
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Contractor's Report 16.4-18

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MEMORANDUM ON THE POLARIZATION-OPTICS
OF THE PHOTOELASTIC SHUTTER

This report discusses the theoretical aspects of the problem of the photoelastic shutter developed under contract OEMsr-576 at the Massachusetts Institute of Technology. In particular it is an investigation of the polarization-optical systems used in connection with this shutter and serves to determine the most favorable system to be used for attaining maximum range of transmission.

The light transmitted by the shutter is in general a mixture of natural light and elliptically polarized light, i.e. so-called partially elliptically (abbrev. pe), or eventually partially linear[1] (pl), or partially circular (pc) polarized light. This is true even if the shutter is illuminated with monochromatic linear (l), elliptical (e), or circular (c) polarized light. Since the mathematical tools necessary for the handling of problems involving partially polarized (pp) light appear to be little known [2], we start our discussion with a short exposition of these mathematical methods.

Part A. POLARIZATION OPTICS.

1) Poincaré's representation of polarized light.

A polarized light beam is usually described by giving its electrical vector

$$E_x = p_1 \cos(\omega t + \varphi_1)$$
$$E_y = p_2 \cos(\omega t + \varphi_2) \qquad (1)$$

referred to a system of axes with the z axes along the direction of propagation. $\varphi_1 = \frac{2\pi z}{\lambda} + \delta_1$, $\varphi_2 = \frac{2\pi z}{\lambda} + \delta_2$ are the phase

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1) We prefer "linear polarized light" over the more commonly used notation "plane polarized light."

2) The writer knows of no textbook in which these mathematical methods are discussed. Certain aspects of the method discussed here are given by F. Perrin, Journal of Chemical Physics <u>10</u> 415 (1942) where older references may be found.

angles, $\nu = \frac{\omega}{2\pi}$ the frequency, p1, p2 the amplitudes. Since the phase angles φ_1 and φ_2 are not observable quantities, it is sufficient for the characterization of the state of polarization of a light beam to give three quantities, namely, p1, p2 and $\varphi_1 - \varphi_2 = \delta_1 - \delta_2 = \delta$. When either one of the three parameters is zero the light is plane polarized; $p_1 = p_2$, $\delta = \pm \pi/2$ are the parameters for right (+) and left (−) circular polarization.

Since p1 p2 and δ change with a change of the orientation of the x, y axes, they are not convenient for a mathematical analysis. A better set of parameters is obtained from the ellipse which is the locus of the end points of the vector E. If the principal axes x', y' of the ellipse are chosen as reference axes E is given by the equation

$$E_{x'} = a \cos \omega t$$
$$E_{y'} = \pm b \sin \omega t \qquad (2)$$

where +b refers to "right" and −b to "left" elliptical polarization. A new set of parameters is therefore obtained by giving:

a = half the major axis. This is to be considered an absolute quantity.

$\pm b$ = half the minor axis $|b| \leq a$

and

α = the angle between the direction of the major axis and the reference axis x. $0 \leq \alpha < \pi$.

A third set can be obtained by introducing the intensity of the light beam. The intensity is given by Pointing's vector $S = \frac{c}{4\pi} (\overline{E_x^2} + \overline{E_y^2})$. Since we are interested only in intensity ratios, it is sufficient to characterize the intensity by

$$I = \frac{8\pi S}{c} = p_1^2 + p_2^2 = a^2 + b^2. \qquad (3)$$

As the other two parameters one choses α, as defined above, and the ratio b/a, or, better, one introduces the "ellipticity" β defined by

$$\tan \beta = \pm b/a, \text{ where } -45° \leq \beta < \pm 45° \qquad (4)$$

The parameters I, α, β lead to <u>Poincaré's representation</u> of polarized light. Since $I > 0$, $0 \leq \alpha < \pi$, $-\frac{\pi}{4} \leq \beta \leq +\pi/4$

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the state of polarization of any polarized light beam can be represented by a vector of length I whose directions are given by the angles 2α and 2β, as shown in diagram 1, where 2α is the longitude and 2β the equatorial distance.

If light undergoes a change of polarization, without a simultaneous change of intensity, f.i., when it goes through a transparent crystal or a sugar solution, the representative point P moves along the surface of a sphere, the so-called Poincaré sphere (abbr. P-sphere). Any point on the equator ($\beta = 0$) of this sphere represents (l) light, the North pole is right (c), the South pole left (e) and generally any point on the northern hemisphere represents right (c) light, and the southern hemisphere corresponds to left (e) light.

2. Poincaré representation of the action of simple optical instruments.

The action of any polarization optical instrument which changes polarized light into other polarized (but not into partially polarized) light can now be described by stating how it influences the representative vector. The effects associated with these instruments are:

Optical Activity (Sugar solution, Quartz with light parallel to axis, Faraday effect). If χ notates the angle of rotation $[\chi = \frac{\pi d}{\lambda}(n_r - n_l)]$ of the optically active substance, it changes the position of any representative Point P along the sphere to a position P' obtained by rotation of the P sphere about its polar axis NS by an angle 2χ. See fig. 2.

Birefringence (parallel crystal plate, Kerr cell, Photoelasticity)

To describe the action of a doubly refracting plate one must give:

1) The orientation φ, i.e., the angle which the optical axis of the plate (axis of the ellipse obtained by intersection with the index ellipsoid) forms with the reference axis X.

2) The phase shift between the ordinary and extraordinary light component created by the plate, $\psi = \frac{2\pi}{\lambda} d(n_o - n_e)$, where d is the thickness of the plate.

Since no intensity change is involved (neglecting reflection) the representative point moves along the P sphere. Its end position P' can be found by the following construction: Rotate the P-sphere by an angle ψ around an axis in the equatorial plane which forms an angle 2φ with the reference axis. See diagram 2.

Polarizer (Nichol, Polaroid)

If the axis of the Polarizer forms an angle α_p with the reference axis, the transmitted (l) light is represented by a vector in the equatorial plane which is at an angle $2\alpha_p$ to the x axis. The length of this vector I_p is found as follows: Construct the normal projection M_p of I on the given direction of I_p. See fig. 2. Then

$$I_p = \tfrac{1}{2} (I + M_p) \qquad (5)$$

(Note that M_p is an algebraic quantity.)

The validity of these rules may readily be checked by verification of certain simple consequences, as for example:

A quarter-wave plate, for which $\psi = 90°$, transforms (l) into (c) light when its axis is at 45° to the plane of polarization.

Any light can be transformed into (l) light by a $\lambda/4$ plate. To this purpose set the axes of the plate parallel to the axes of the ellipse ($\alpha = \varphi$) and the transmitted (l) light is oriented at an angle β to these axes. (Sénarmont Compensator)

When using optically active and birefringent plates, or several of the latter, in succession, the character of the transmitted light depends on the succession of the plates, because the addition of finite rotations is not commutative.

The intensity of light transmitted through a polarizer is a maximum when its axis is parallel to the major axis of the incident (e) light ($\alpha_p = \alpha$). It is a minimum when $\alpha_p = \alpha + 90$ and varies with the orientation α_p according to

$$I_p = \tfrac{1}{2} \left[I + M_{max} \cos 2(\alpha_p - \alpha) \right]$$
$$= \tfrac{1}{2} I \left[1 + \cos\beta \cos 2(\alpha_p - \alpha) \right] \qquad (6)$$

A birefringent plate shall be called a phase plate or ψ-plate, its optical axis is called "axis of the ψ-plate."

3) A variable "Quarter-wave" plate.

The importance of $\lambda/4$ plates, i.e. $\pi/2$-plates, in polarization optical investigations arises from the fact that they serve to transform any kind of (c) light into (l) light, and vice versa. All $\lambda/4$ plates have for practical purposes the disadvantage of being serviceable for only one wavelength of light, because ψ varies with λ. For instance the commonly used $\lambda/4$ plates are correct usually for $\lambda \sim 6000A$. For infrared light of $\lambda = 9000A$ they are approximately $\pi/3$-plates and for ultra-violet light of $\lambda = 4000$ they are $3\pi/4$-plates.

From Poincaré's representation it is apparant that whatever can be accomplished with a $\lambda/4$ plate, can also be obtained by using two or more arbitrary ψ-plates in succession. For instance the transformation of (l) into (c) light can be accomplished by two ψ plates, provided $\pi/4 \leq \psi \leq 3\pi/4$. Thus two "visible" $\lambda/4$ plates can perform the (l)-(c) transformation for light of any wavelength between $\lambda = 12000$ to $\lambda = 4000A$. By adjusting the angles of these plates relative to the orientation of the (l) light a "Quarter-wave" plate for any kind of light within this range may be constructed. The directions for this construction can easily be derived from the Poincaré sphere as follows (see diagram 3). ?

Let P on the equator represent the incident (l) light. If the first ψ-plate is oriented at an angle φ_1 to the orientation of the (l) light, it shifts the point P to P' by a rotation ψ around A, where AP = AP' = $2\varphi_1$. To attain (c) light a second identical ψ-plate is introduced. If its axis forms an angle φ_2 with that of the first ψ-plate, it creates a rotation ψ around B, where AB = $2\varphi_2$. This rotation shifts P' to P", and the transmitted light is (c) polarized if P" is at the Pole, i.e. if BP" = BP' = 90°, which implies also that P"BA = 90°. The spherical triangle AP'B gives sin law:

$$\frac{\sin 2\varphi_1}{\sin 90^\circ} = \frac{\sin(90-\psi)}{\sin(180-\psi)}$$

$$\text{or} \quad \sin 2\varphi_1 = \cot g\, \psi \qquad (7)$$

Cos-law:

$$\cos 90 = \cos 2\varphi_1 \cos 2\varphi_2 - \sin 2\varphi_1 \sin 2\varphi_2 \cos \psi$$

$$\cos 2\varphi_1 = \cos 90 \cos 2\varphi_2 + \sin 90 \sin 2\varphi_2 \sin \psi$$

whence

$$\tan 2\varphi_2 = \frac{\cot g\, 2\varphi_1}{\cos \psi}$$

$$\sin 2\varphi_2 = \frac{\cos 2\varphi_1}{\sin \psi}$$

and, using (7) one obtains

$$\cos 2\varphi_2 = \text{cotg}^2\, \psi \qquad (8)$$

Equations (7) and (8) determine the orientation of both plates when the phase shift ψ of the plates are known for the particular wavelength used. The resulting values of φ_1 and $(\varphi_1+\varphi_2)$, the latter giving the orientation of the second ψ-plate with respect to the (1) light, are plotted in Fig. 4. Note that $(\varphi_1+\varphi_2)$ has a maximum for $\psi = 60°$. Hence slight changes of the wavelength or of ψ do not affect the orientation of the second plate, and only that of the first plate is critical. This result is of importance when two "visible" Polaroids are used to make a $\lambda/4$ plate for infrared light.

It is important to note that this double plate, although it transforms (1) light into (c) light, has properties which differ from that of $\lambda/4$ plate. Consider f.i. (1) light polarized at 45° to that represented by P. This light would not be changed by an ordinary $\lambda/4$ plate. The double ψ-plate, however, changes the polarization of this light, because two rotations around different axes leave the position of no point on the P-sphere unaltered. In our example the incident light is represented by point Q in diagram 3, where PQ = 90°. The first ψ-plate shifts this point to Q' and the second to Q". Q" is again on the equator, i.e. the transmitted light is (1) polarized, because P"Q" must be 90° and P" is at the pole.

A simple calculation, using the fact that AQ = AQ' = BQ' = BQ" = $2\varphi_1 + 90$, shows that the Q-light is rotated by an angle $\gamma = \frac{QQ"}{2}$

$$\gamma = 90° - \left[\varphi_1 + (\varphi_1+\varphi_2)\right] \qquad (9)$$

Tests with double ψ-plates have verified the above conclusions.

In passing it may be noted that with three ψ-plates it should be possible to construct multiple ψ-plates which have all the properties of $\lambda/4$ plates.

4) Stockes' representation of partially polarized light.

Stockes has shown that any arbitrary monochromatic light, f.i. as obtained by the superposition of an arbitrary number of polarized, coherent or incoherent, components, can always be considered as a superposition of natural light (e.i. for which E assumes all directions and phases with the same probability) and elliptically polarized light. Thus the total intensity I of any (p.p) light can be decomposed into

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$$I = I_N + I_p \qquad (10)$$

where I_N and I_p are the intensities of the natural and the polarized components. The polarized component in turn can be represented by Poincare's representation by three parameters I_p, 2α, 2β. Thus the representation of (p.p) light requires 4 parameters. While I_N, I_p, α and β are the set with a simple physical significance, they are not those which are suitable for calculations.

To arrive at a more suitable set we consider the problem of the superposition of two (p.p) light components. Let the first component be characterized by I'_N, I'_p, α', β', $I' = I'_N + I'_p$, the second component by I''_N, I''_p, α'', β'', $I'' = I''_N + I''_p$ and the result of the superposition by I_N, I_p, α, β, $I = I_N + I_p$. If the two components are coherent the addition gives obviously $I_N = I'_N + I''_N$, $I_p = I'_p + I''_p$, but the values of α and β cannot be given because they depend on the phase relation between the components. The superposition is quite different when the two components are incoherent. In this case $I_N \gtreqless I'_N + I''_N$, because the superposition of two incoherent polarized components will in general produce a natural component. (Example: Two (l) components polarized in different directions.) Thus it is seen that I_N and I_p are not additive parameters for incoherent superposition, however, since $I = I' + I''$ will always hold, we find that I is an additive parameter and therefore more suitable than I_N and I_p for the representation of (p.p) light. To find the other suitable parameters it is necessary to determine the polarized component of the resultant of incoherent superposition. A somewhat lengthy but elementary calculation leads to the simple result that in Poincaré's representation I_p is obtained by vector addition of the vectors I'_p and I''_p. This result implies naturally that the cartesian coordinates of the representative points P' and P" are additive q uantities for incoherent superposition. The suitable parameters for (p.p) light are therefore, see fig. 1

$$\begin{aligned} I &= I_N + I_p \\ M &= I_p \cos 2\beta \cos 2\alpha = p_1{}^2 - p_2{}^2 \\ C &= I_p \cos 2\beta \sin 2\alpha = 2p_1p_2\cos\delta \\ S &= I_p \sin 2\beta = 2\, p_1p_2\sin\delta \end{aligned} \qquad (11)$$

and the addition of incoherent components gives

$$I = I' + I''$$

$$M = M' + M''$$

$$C = C' + C''$$

$$S = S' + S''$$

The four parameters are called Stockes' parameter (abbr. St-P). Note that I and S are independent of choice of the reference axis x = M. A rotation of the reference axis by an angle λ changes M and C to

$$\begin{aligned} M^* &= M \cos 2\lambda + C \sin 2\lambda \\ C^* &= -M \sin 2\lambda + C \cos 2\lambda \end{aligned} \qquad (12)$$

From the 4 St-P all information concerning the natural and polarized components can be derived by means of the relations

$$I_p = +\sqrt{M^2 + C^2 + S^2} = p_1^{\,2} + p_2^{\,2}$$

$$I_N = I - I_p \qquad\qquad p_1^2 = \tfrac{1}{2}(I_p + M)$$

$$\tan 2\alpha = C/M \qquad\qquad p_2^{\,2} = \tfrac{1}{2}(I_p - M)$$

$$\sin 2\beta = S/I_p \qquad\qquad \tan\delta = S/C$$

and the following implications are apparent:

S = 0 implies (pl) light.

C = 0 implies that the axis of the ellipse representing the polarized component is parallel or normal to the reference axis X.

M = 0 implies that this axis is at 45° to X.

M = C = 0 implies (pc) light.

M = C = S = 0 implies natural light.

5) The superposition of incoherent components.

The law of additivity of the St-P can of course be extended to arbitrary many incoherent (pp) components

$$\begin{aligned} I &= \sum I_i \qquad & C &= \sum C_i \\ M &= \sum M_i \qquad & S &= \sum S_i \end{aligned} \qquad (14)$$

and can be put in integral form as illustrated by the following example:

Consider a plate of area A which emits light. Assume that the intensity and state of polarization varies continuously over the area. The light coming from an element $dA = dxdy$ can then be characterized by

$$dI = j(x,y)\ dxdy$$

$$dM = j_p(x,y)\ \cos 2\beta(x,y)\ \cos 2\alpha(x,y)\ dxdy \quad = m(x,y)\ dxdy$$

$$dC = j_p(x,y)\ \cos 2\beta(x,y)\ \sin 2\alpha(x,y)\ dxdy \quad = c(x,y)\ dxdy$$

$$dS = j_p(x,y)\ \sin 2\beta(x,y)\ dxdy \quad = s(x,y)\ dxdy$$

and if all contributions are incoherent the addition law states

$$I = \int j dA \qquad C = \int c dA \qquad (15)$$
$$M = \int m dA \qquad S = \int s dA$$

The question whether two components are coherent or incoherent is sometimes difficult to decide. They are certainly incoherent when they originate in different light sources or when the light paths from the common source differ greatly, also when theoretical reason can be given that the components have slightly different frequencies, even though this difference can not be experimentally shown (f.i. Doppler effect) so that for all practical purposes the two components have the same frequency.

Of special interest for our applications is the following case: Suppose two light beams originate in the same source, pass through different instruments and emerge differently polarized although the lengths of the two paths is the same. It can be shown that these beams can be considered to be incoherent, provided the aperture stops in both beams are sufficiently large so that the path of both beams can be described in terms of geometrical optics. The situation can be explained by considering a modification of Young's experiment for the diffraction of light in which the two light waves from two coherent sources pass through different polarizing devices. In the arrangement shown in fig. 5, the light from the two sources S_1, S_2 are made (1) polarized, vertical and horizontal, respectively, by the two Polaroids P_1 and P_2. Without these polarizers the image on the screen Sc is the well-known diffraction pattern with maxima at equidistant points A_i, where the two waves are in phase. With the polarizers inserted the diffraction pattern disappears, but due to the phase relations we conclude that at the positions A_i the resulting light will be (l) polarized at 45°. At points B_i halfway between A_i, where the minima would be, the resulting light is (l) polarized at -45°, and at intermediate points it is (e) polarized. Thus the diffraction pattern could again be made visible by observing the screen through a third polarizer at 45° to P_1 and P_2. Now if the slits are opened the screen shows an image of the primary source S and the light in this image is the superposition of all states of polarization, which in this case gives natural light. Hence in the framework of geometrical optics the two beams S_1 and S_2, although they have a common source, must be considered incoherent.

6) <u>Perrin's representation of the action of polarization-optical instruments</u>.

A beam of light with the St.P. I_1, M_1, C_1, S_1 entering any optical instrument produces transmitted (or reflected, refracted or scattered) light with St.P. I_1', M_1', C_1', S_1'. These may refer either to the emerging beam in a definite direction or to the total (integrated) emerging light. Obviously I_1' M_1', C_1', S_1' are certain functions of I_1, M_1, C_1, S_1 and the action of any optical instrument can be described by giving these four functions. If now a second incoherent beam with the St.P. I_2, M_2, C_2, S_2, which gives rise to a transmitted beam I_2', M_2', C_2', S_2' enters simultaneously with the first beam, we can conclude from the additivity law that, if $I' = f_i(I,M,C,S)$

$$I' = I_1' + I_2' = f_i(I_1,M_1,C_1,S_1) + f_i(I_2,M_2,C_2,S_2) =$$

$$f_i(I,M,C,S) = f_i(I_1+I_2,\ M_1+M_2,\ C_1+C_2,\ S_1+S_2) \tag{16}$$

where f_i is everywhere the same function. Similar equations hold also for f_m, f_c, f_s defined by $M' = f_m(I,M,C,S)$, $C' = f_c(I,M,C,S)$, $S' = f_s(I,M,C,S)$. Eq. (16) can only be satisfied if the function f_i is linear in all parameters. Hence we must have for any optical instrument

$$\begin{aligned} I' &= a_{11} I + a_{12} M + a_{13} C + a_{14} S \\ M' &= a_{21} I + a_{22} M + a_{23} C + a_{24} S \\ C' &= a_{31} I + a_{32} M + a_{33} C + a_{34} S \\ S' &= a_{41} I + a_{42} M + a_{43} C + a_{44} S \end{aligned} \tag{17}$$

where the matrix a_{ik} with 16 coefficients is characteristic for the instrument. a_{ik} is in general dependent on the direction of the incident light and of the direction of observation. a_{ik} are called the Perrin coefficients. It can be shown that for any instrument which satisfied the so-called reciprocity relation $a_{ik} = \pm a_{ki}$. The reciprocity relation implies that when the position of source and observer are exchanged the intensity reaching the observer is the same ("If I can see you, you can see me", holds for most instruments, except for Faraday effect.)

The problem of any polarization optical instrument or effect is therefore solved when the 16 coefficients a_{ik} can be evaluated. It is the purpose of the paper to establish the matrix a_{ik} for the photoelastic shutter, and for instruments used in conjunction with the shutter. The latter are:

ψ **Plate:**

Since the St.P. parameter can readily be transformed by means of Eq . (12) to any new reference axis, it is sufficient to consider the case where the axis of the ψ plate is in the x-direction ($\varphi=0$). As the intensity is not changed by a ψ-plate (neglecting reflections) one has $a_{11} = 1$, $a_{12} = a_{13} = a_{14} = a_{21} = a_{31} = a_{41} = 0$. Since polarized light is modified corresponding to a rotation ψ of the P sphere around the x=M axis one finds

$$\left| a_{ik} \right| = \left| \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\psi & \sin\psi \\ 0 & 0 & -\sin\psi & \cos\psi \end{matrix} \right| \qquad (18)$$

Polarizer:

For a perfect polarizer the transmitted light is (1) polarized, hence $M' = I' \cos 2\alpha_p$, $C' = I' \sin 2\alpha_p$, $S' = 0$ which requires that $a_{4i} = a_{i4} = 0$ and $a_{2i} = a_{1i} \cos 2\alpha_p$, $a_{3i} = a_{1i} \sin 2\alpha_p$. These conditions, and $a_{ik} = a_{ki}$, are sufficient to establish

$$a_{ik} = a_{11} \begin{vmatrix} 1 & \cos 2\alpha_p & \sin 2\alpha_p & 0 \\ \cos 2\alpha_p & \cos^2 2\alpha_p & \sin 2\alpha_p \cos 2\alpha_p & 0 \\ \sin 2\alpha_p & \sin 2\alpha_p \cos 2\alpha_p & \sin^2 2\alpha_p & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} \quad (19)$$

where a_{11} = 1/2 when 50% of natural light is transmitted.

7) <u>The transformation of Polarization changes into Intensity changes</u>.

The state of polarization of a light beam cannot be directly observed. Experimentally it is always necessary to introduce auxiliary equipment into the beam to transform changes of polarization into changes of intensity. As auxiliary equipment we have only to consider ψ-Plates and Polarizers. The latter is always necessary, since it is the only instrument for which the transmitted intensity varies with the state of polarization of the incident light. The problem as it arises in connection with the receiver for the photoelastic shutter may be formulated as follows:

If at the time t_1 the light is characterized by $I_1 = I, M_1, C_1, S_1$ and at a later time t_2 by $I_2 = I, M_2, C_2, S_2$ what should be the orientation of a ψ-plate and the analyzing Polaroid in order to create the largest intensity change in the final receiver (photocell). To answer this problem we ask first: If two beams have the same total intensity I but different polarized components I^*_{p1} and I^*_{p2}, what must be the orientation of I^*_p in the Poincaré representation in order that a perfect Polaroid creates the largest intensity difference? According to (19) the transmitted intensities J are

$$J_1 = \tfrac{1}{2}\left[I + M^*_1 \cos 2\alpha + C^*_1 \sin 2\alpha\right]$$

$$J_2 = \tfrac{1}{2}\left[I + M^*_2 \cos 2\alpha + C^*_2 \sin 2\alpha\right]$$

and the change is

$$\Delta J = J_1 - J_2 = \tfrac{1}{2}\left[(M^*_1 - M^*_2) \cos 2\alpha_p + (C^*_1 - C^*_2) \sin 2\alpha_p\right]$$

Now $(M^*_1 - M^*_2)$, $(C^*_1 - C^*_2)$ are the M and C components of the vector $(I^*_{p1} - I^*_{p2})$, and $\cos 2\alpha_p$ and $\sin 2\alpha_p$ can be interpreted as the M and C component of a unit vector $I_{\alpha p}$ in the equatorial plane. Hence $2\Delta J$ can be interpreted as the scalar product

$$2 \cdot \Delta J = \vec{I}_{\alpha p} \cdot (I_{p1}^{*} - I_{p2}^{*}) \qquad (20)$$

It follows that ΔJ is a maximum when $(I_{p1}^{*}-I_{p2}^{*})$ is parallel to $\vec{I}_{\alpha p}$, e.i. $(I_{p1}^{*}-I_{p2}^{*})$ must be parallel to the equatorial plane, and the axis of the polaroid is to be adjusted parallel to $(I_{p1}^{*}-I_{p2}^{*})$.

Now if the polarized components of the light at t_1 and t_2 do not satisfy the above condition for $(I_{p1}^{*}-I_{p2}^{*})$ they can be changed by a ψ-plate so that the condition is satisfied when the light enters the Polaroid. According to eq. 18 and section 1 a ψ-plate rotates $\vec{I}_{p1}$ and $\vec{I}_{p2}$ around an axis in the equatorial plane by an angle of ψ, without altering the magnitude of I, I_{p1} and I_{p2}, and it is obviously always possible to find a ψ-plate so that $\vec{I}_{p1}-\vec{I}_{p2}$ is after the rotation parallel to the equatorial plane.

The answer to our problem is therefore: φ and ψ of the ψ-plate must be adjusted so that the vector $(I_{p1}-I_{p2})$ becomes parallel to the equatorial plane, and the Polaroid is to be adjusted with its axis parallel (or normal) to the resulting direction of $(I_{p1}-I_{p2})$ after rotation. From eq. (20) it follows that the maximum change of intensity which can be produced is

$$\Delta J_{max} = \frac{1}{2}\left[(M_1-M_2)^2 + (C_1-C_2)^2 + (S_1-S_2)^2\right]^{1/2} \qquad (21)$$

Although ΔJ_{max} depends only on the polarized component of the light, this does not mean that the validity of (21) is restricted to cases where only I_p is altered. The only restriction is that the total intensity is not changing with time.

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Part B. THE PHOTOELASTIC SHUTTER.

8) Optical characteristics of the shutter.

The shutter as described in a previous report, is a rectangular or square plate of glass in which a standing elastic wave is set up by means of piezo-electric Quartz crystals. In general the elastic wave has three sets of nodal planes parallel to the faces of the plate, but the frequency can be adjusted so that only one set of nodes appear. The more general case of several sets of nodes is left for a later discussion. We consider here only the case where the strains in the glass can be described by

$$X_x = A \sin 2\pi \frac{x}{\lambda_s} \sin \omega_s T$$

$$Y_y = Z_2 = X_y = Z_x = Y_z = 0.$$

i.e., a standing wave with nodes parallel to y. The light passes through the plate in the z direction. λ_s and $V_s = \frac{\omega_s}{2\pi}$ are the wavelength and frequency of the sound wave; in current experiments V_s is about 1 megacycle and λ_s about 0.5 cm.

The photoelastic effect of the strain X_x is given by

$$n_x = n + u \sin 2\pi \frac{x}{\lambda_s} \sin \omega_s t$$

$$n_y = n + v \sin 2\pi \frac{x}{\lambda_s} \sin \omega_s t \qquad (22)$$

where n_x, n_y are the principal refractive indices, n the refractive index of the glass, $u = n^2 q A$, $v = n^2 q A$, where p, q are Neumann's strain-optical constants of the glass. The shutter is therefore a ψ-plate with a variable phase shift

$$\psi = (\psi_m) \sin 2\pi \frac{x}{\lambda_s} \sin \omega_s t \qquad (23)$$

where

$$\psi_m = \frac{2\pi}{\lambda} d (u-v) \qquad (24)$$

d being the thickness of the plate.

In the following discussions we will neglect the time variation of n_x, n_y and ψ, i.e. we consider first the stationary case and replace later the constant values u, v,

ψ_m by periodically varying functions. This procedure involves an approximation, because it is known that the optical effect of a standing supersonic wave is not a simple superposition of the effects of two simple waves traveling in opposite directions. However, the exact solution is for all practical cases sufficiently close to our approximation that it seems unnecessary to present here the rather elaborate exact solution.

The optical problem of the shutter can be discussed by two entirely different methods which are presented in the succeeding section.

9) <u>The diffraction method</u>.

We assume that the entire area A of the shutter is illuminated uniformly at normal incidence by a plane light wave with an arbitrary ellipticity, i.e. the incident light is characterized by

$$E_x = p_1 \cos (\omega t + \varphi_1)$$
$$E_y = p_2 \cos (\omega t + \varphi_2) \qquad (25)$$

or by the St.-P.

$$I = p_1^2 + p_2^2$$
$$M = p_1^2 - p_2^2$$
$$C = 2p_1p_2 \cos (\varphi_1 - \varphi_2) \qquad (26)$$
$$S = 2p_1p_2 \sin (\varphi_1 - \varphi_2)$$

Using the first representation (25) we can consider the two waves E_x and E_y separately. Each of these waves suffers a phase shift when it passes through the glass, namely for E_x

$$\Delta\varphi_1 = \frac{2\pi d}{\lambda} (n + u \sin 2\pi \frac{x}{\lambda_s})$$

Since $\Delta\varphi_1$ varies with x the transmitted wave is no longer plane, rather it has a "corrugated" wave front. The shutter is a "phase-grating" for each light component. Raman and Nath have shown that the "corrugated" wave can be decomposed into a set of plane waves traveling in different directions. These are the diffraction orders. As is the case for an ordinary grating with a period λ_s the diffraction angles are given by

$$\sin \gamma_i = i \frac{\lambda}{\lambda_s} \quad (i=0, \pm 1, \pm 2....) \qquad (27)$$

and the amplitudes of the diffracted waves are found to be

$$E_x^{\,i} = p_1 J_i (\psi_1) \qquad \psi_1 = \frac{2\pi d}{\lambda} u$$

$$E_y^{\,i} = p_2 J_i (\psi_2) \qquad \psi_2 = \frac{2\pi d}{\lambda} v \qquad (28)$$

where J_i denotes the Bessel function of i'th order.

According to this consideration the shutter works essentially on the same principle as the supersonic shutter in the Scofoni system. In this system supersonics of high frequency are used so that the diffraction angles become large. The receiver will then collect only the light from a single diffraction order and periodic variations of ψ_1 or ψ_2 produce modulation of the light intensity of these orders. Our photoelastic shutter, when operated at frequencies of about 50 megacycles could be operated with natural light, provided the light cone has an angle of aperture smaller than the 1st diffraction angle. For a cone of 10° total aperture supersonics of frequency 4.10^8 would be required, Obviously this is not practical. In the photoelastic shutter one makes use of the fact that the various diffraction orders are differently polarized. Instead of a spacial separation of the orders used in the Scofony system, a separation by polarization is carried out. This carries the advantage that supersonics of low frequencies can be used, though it has the disadvantage of incomplete separation, The latter is offset by the fact that stronger supersonics can be produced at low frequencies.

In the photoelastic shutter the transmitted light is a superposition of all diffraction orders because the diffraction angle (of about 1/2' in our experiments) is negligible compared to the width of the light cone of 7°.

In carrying out the addition of all diffraction orders it should be noted that the E_x and E_y components are necessarily coherent, and because they have the same optical path their phase difference is the same as in the incident light. However, the light in the various diffraction orders is incoherent. While this is not obvious for the stationary case, it must be so for the dynamic case, because the shutter modulates the light in the different orders by a different frequency (i.e. different harmonics of v_s). The frequency shift is much too small to be observed ($v_s/v = 3 \cdot 10^{-9}$, but is sufficient to guaranty incoherency.

It follows that the St-P. of each order can be computed in the same manner as for the incident light

$$I_i = p_1^2 J_i^2(\psi_1) + p_2^2 J_i^2(\psi_2)$$

$$M_i = p_1^2 J_i^2(\psi_1) - p_2^2 J_i^2(\psi_2)$$

$$C_i = 2p_1 p_2 J_i(\psi_1) J_i(\psi_2) \cos(\varphi_1 - \varphi_2) \qquad (29)$$

$$S_i = 2p_1 p_2 J_i(\psi_1) J_i(\psi_2) \sin(\varphi_1 - \varphi_2)$$

and addition of all orders gives for the transmitted light

$$I' = \sum I_i \qquad C' = \sum C_i$$

$$M' = \sum M_i \qquad S' = \sum S_i$$

These sums take a simple form on account of the following relations for Bessel functions:

$$\sum_{i=-\infty}^{i=+\infty} J_i^2(x) = 1$$

$$\sum_{i=-\infty}^{i=+\infty} J_i(x) J_i(y) = J_o(x-y)$$

[for the latter relation see Whittaker & Watson, Modern Analysis, p. 380.]

Taking into account (26) and noting that according to (28) and (24)

$$\psi_1 - \psi_2 = \psi_m \qquad (30)$$

one finds as final result

$$\begin{aligned} I' &= I \\ M' &= M \\ C' &= KC \qquad K = J_o(\psi_m) \qquad (31) \\ S' &= KS \end{aligned}$$

10) <u>The Polarization method.</u>

In carrying out this method we divide the shutter in small strips of width dx and consider each strip as a phase plate with its axis parallel to x and a phase shift $\psi = \psi_m \sin \frac{2\pi x}{\lambda_s}$. If I, M, C, S are the Stockes parameter of the incident light, which is assumed to be uniform over the whole area of the shutter, then the light falling on the strip dx is

$$\begin{aligned} dI &= j dx \qquad & j &= I/W \\ dM &= m dx \qquad & m &= M/W \\ dC &= c dx \qquad & c &= C/W \qquad (32) \\ dS &= s dx \qquad & s &= S/W \end{aligned}$$

W is the width of the shutter in the x-direction and can be assumed, without loss of generality, to be a whole multiple of λ_s.

According to eq..(18) the light transmitted through the narrow strip has the St-P.

$$\begin{aligned} dI' &= dI \\ dM' &= dM \\ dC' &= dC \cdot \cos\psi + dS \sin\psi \qquad (33) \\ dS' &= dC \quad \sin\psi + dS \cos\psi \end{aligned}$$

Following the argument of section 5, these contributions are to be considered incoherent, hence the transmitted light is

$$I' = \int j dx = I$$

$$M' = \int m dx = M$$

$$C' = C \frac{1}{W} \int_0^W \cos(\psi_m \sin 2\pi\frac{x}{\lambda_s}) dx + S \frac{1}{W}\int_0^W \sin(\psi_m \sin 2\pi\frac{x}{\lambda_s}) dx \qquad (34)$$

$$S' = -C \frac{1}{W}\int_0^W \sin(\psi_m \sin' 2\pi\frac{x}{\lambda_s}) dx + S \frac{1}{W}\int_0^W \cos(\psi_m \sin 2\pi\frac{x}{\lambda_s}) dx$$

From the periodicity of the integrand it follows for both integrals that $\frac{I}{W}\int_0^W = \frac{1}{\lambda_s}\int_0^{\lambda_s}$. The substitution $2\pi\frac{x}{\lambda_s} = z$ transforms this into

$$\frac{1}{2\pi}\int_0^{2\pi} \cos(\psi_m \sin z) dz \quad \text{and} \quad \frac{1}{2\pi}\int_0^{2\pi} \sin(\psi_m \sin z) dz$$

respectively. The first integral is the well-known integral representation of $J_0(\psi_m)$. The second integral is zero, because $\int_0^{\pi} = -\int_{\pi}^{2\pi}$. Hence the final result

$$I' = I \qquad C' = KC$$
$$M' = M \qquad S' = KS \qquad K = J_0(\psi_m) \qquad (35)$$

is identical with that derived by the diffraction method.

11) <u>The properties of the uniform shutter.</u>

The shutter discussed in the preceding section shall be called "uniform" to indicate that the entire area is illuminated with light of the same characteristics. The matrix of the Perrin coefficients of this shutter are

$$a_{ik} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & K & 0 \\ 0 & 0 & 0 & K \end{vmatrix} \qquad (36)$$

and the following conclusions are elementary:

a) Natural light (I,0,0,0) and (1) or (p,1) light (I,M,0,0) vibrating parallel or normal to x are not changed by the shutter.

b) Since for practical reasons the incident light is polarized (and not p p), whence $I^2 = M^2 + C^2 + S^2$, and since the M component remains unaltered, the shutter will give the best results when C and S are as large as possible. This requires M = 0, and means that the axis of the incident light is to be oriented at 45° to x (or to the nodes). Any elliptically polarized light of this character is transformed into partially elliptical light with the axes remaining at 45°. The natural component has the intensity $I_N = I(1-K)$, the polarized component $I_p = KI$, the ellipticity of the latter is the same as for incident light ($C'/S' = C/S$)

When the shutter is not operated, $\psi_m = 0$, and $K = J_0(0) = 1$. Application of a stress creates a change of C' from C to KC and of S' from S to KS. As shown in section 7 the largest possible intensity variation which the stress can create - using the most effective analyzer - is therefore

$$\Delta J_{max} = \tfrac{1}{2}(1-K)\left[C^2+S^2\right]^{1/2} = \tfrac{1}{2} I\,(1-K) \qquad (37)$$

and is entirely independent of whether the incident light is made elliptical or not. The most simple and at the same time the most effective method consists therefore in using (1) incident light, polarized at 45°, e.i. M = S = 0, C = I, The transmitted light with M' = S' = 0, C = KI is then (p,1) polarized, and since its I_p vector is in the equator a simple Polaroid, parallel or crossed, is the best analyzer. Thus the shutter operates best between crossed or parallel Polaroids at 45° to x. A phase plate can be inserted in front of the shutter, but then a compensating plate (with the same ψ, but axis rotated by 90°) must be used in front of the analyzer. Except for additional reflection losses, the latter method gives the same result as simple Polaroids. The use of phase plates is recommended only when additional secrecy is required.

12) Frequency characteristics of uniform shutter.

a) For C W transmission

We introduce now the time variation of ψ_m by replacing it by $\psi_m \sin \omega_s t$. The light transmitted by the uniform shutter is now

$$j = \frac{I_t}{I_o} = \tfrac{1}{2}\left[1 - J_o(\psi_m \sin \omega_s t)\right] \qquad (38)$$

Where I_o would be the incident light falling on the shutter if reflection losses were negligible and if the polaroids would transmit 50% of natural light. The corrections necessary for I_o will be taken into account later.

The function $1-J_o(x) = (\frac{x}{2})^2 - \frac{1}{(2!)^2}(\frac{x}{2})^4 + \frac{1}{(3!)^2}(\frac{x}{2})^6 \ldots$ is an even function. Hence the Fourier development of j will contain only the frequencies $2N\omega_s$, i.e. the radio receiver must be tuned to one of the even harmonics of the frequency of the sender. To determine the strength of the various harmonics we set:

$$\tfrac{1}{2}\left[1-J_o(\psi_m \sin \omega_s t)\right] = K_o(\psi_m) - \sum R_{2n}(\psi_m) \cos 2n\omega_s t \qquad (39)$$

From Fourier's theorems we obtain

$$K_o(\psi_m) = \frac{V_s}{2}\int_0^{1/V_s}\left[1 - J_o(\psi_m \sin 2\pi V_s t)\right] dt \qquad (40)$$

K_o represents the average constant light intensity passing the shutter during C.W. transmission. The strength of the radio components are given by

$$R_{2n}(\psi_m) = 2\,\frac{V_s}{2}\int_0^{1/V_s}\left[1-J_o(\psi_m \sin 2\pi V_s t)\right] \cos 4\pi n V_s t \; dt \qquad (41)$$

Substituting $2\pi V_s t = z$ and noting that $\int_0^{2\pi} = 2\int_0^{\pi}$ one finds

$$K_o(\psi_m) = \tfrac{1}{2}\left[1 - \frac{1}{\pi}\int_0^{\pi} J_o(\psi_m \sin z)\, dz\right]$$

$$R_{2n}(\psi_m) = +\frac{1}{\pi}\int_0^{\pi} J_o(\psi_m \sin z) \cos(2nz)\, dz$$

The integrals can be simply expressed, giving

$$K_o = \frac{1}{2}\left[1 - J_o^{\,2}\left(\frac{\psi_m}{2}\right)\right] \qquad (42)$$

$$R_{2n} = J_n^{\,2}\left(\frac{\psi_m}{2}\right) \qquad (43)$$

(See Watson: Theory of Bessel Functions, Cambridge 1922, p.32.)

A discussion of these functions is given in the concluding section of the report. See Fig. 9.

b) The α measurements.

The experimental evaluation of ψ_m in a vibrating block could be carried out with suitable compensators. A much easier method is based on the fact that in the uniform shutter the average transmitted intensity is given by $I_t = I_o K_o(\psi_m)$. Now this intensity can be compared, without altering the optical system and hence without a change of the correction factors of I_o, with that transmitted through an analyzer which is deviated by an angle α from the "crossed" position. Since this intensity is $I_t' = I_o \sin^2 \alpha$, it follows that when $I_t = I_t'$, $K_o(\psi_m) = \sin^2\alpha$. Thus measurements of the "uncrossing" angle α for which the transmission through the nonvibrating shutter is the same as through the vibrating shutter with crossed Polaroids permits a determination of ψ_m. This method is particularly convenient for NAN, where compensator measurements are difficult. The correlation between α and ψ_m is plotted in Fig. 6.

For experimental purposes it is more convenient to correlate observable quantities. We have therefore adopted in Figs. 9 and 10 the representation of the various functions in dependance of α rather than of ψ. The corresponding values of ψ are given in the scale on top of the figures. Since in the most important range for $\psi < 3$, α and ψ are nearly proportional the essential features of the $K_o(\psi_m)$ and the other curves are retained in the $K_o(\alpha)$ etc. plots.

c) MCW and voice transmission.

In this type of transmission the peak value ψ_m of the supersonic is varied periodically. One replaces therfore ψ_m by $\psi_m(1 + r \sin \omega_\mu t)$ where 100 r [$r<1$] is the percentage modulation of the supersonic wave in the shutter. The variation of ψ_m creates a periodic variation of the average value

K_o and of the r.f. modulated parts R_{2m} of the light. The modulation of K_o gives rise to a component which can be registered with an audio amplifier, while the modulation of R_{2n} gives a modulated radio output of the photocell.

To find the modulated output it would be necessary to carry out a Fourier analysis of $K_o[\psi_m(1+r \sin \omega_A t)]$ and $R_{2m}[\psi_m(1+r \sin \omega_A t)]$. This turns out to lead to unmanageable expressions. A satisfactory approximation, valid for small values of r, is obtained with the development:

$$K_o\left[\psi_m(1+r \sin \omega_A t)\right] = K_o(\psi_m) + \left(\frac{\partial K_o}{\partial \psi_m}\right) r\,\psi_m \sin \omega_A t + \ldots\ldots$$

The Fourier analysis

$$K_o\left[\psi_m(1+r \sin \omega_A t)\right. = K_o{}^o(\psi_m) + \sum r A_o{}^n \sin n\omega t$$

furnishes now for the amplitude A'_o of the modulated component (referred to 100% modulation)

$$A_o{}' = \psi_m \frac{\partial K_o}{\partial \psi_m} + \frac{r^2}{6} \psi_m^3 \, \frac{3}{4} \, \frac{\partial^3 K_o}{\partial \psi_m{}^3} + \ldots\ldots\ldots$$

As seen from Fig. 9 K_o and R_2 are over a large range of ψ_m nearly linear. Thus for practical purposes it is sufficient to take

$$A_o{}' = \psi_m \frac{\partial K_o}{\partial \psi_m} \qquad (44)$$

A similar development for $R_{2n}[\psi_m(1+r \sin \omega_A t)]$ gives a radio modulated output with a response to the fundamental of ω_A (referred to r = 1)

$$A_2{}' = \psi_m \frac{\partial R_2}{\partial \psi_m} \qquad (45)$$

These expressions can be represented in terms of Bessel functions

$$A_o{}' = \frac{\psi}{2} J_o(\psi/2)\, J_1(\psi/2) \qquad (44')$$

$$A_2{}' = 2J_1(\psi/2)\left[J_o(\psi/2)\,\frac{\psi}{2} - J_1(\psi/2)\right] \qquad (45')$$

These functions are plotted in Fig. 10, and the implications are discussed in the concluding section.

The chief features of the uniform shutter may be summarized as follows:

a) It permits C.W. transmission, but the receiver must be tuned to twice the frequency of the sender. For other even harmonics the transmission is weak.

b) M.C.W. and voice transmission can be obtained either by audio reception or R.F. reception at the double frequency.

c) The light reaching the receiver is always either (l) or strongly (pl) polarized, depending on whether the analyzer is placed at the sender or receiver.

13) The "stripped" shutter.

The uniform shutter is not the most effective method for transforming elastic vibration in the block into variation of light intensity. To realize this fact we consider the contribution of an infinitely narrow strip of the shutter. According to (33) a change from $\psi = 0$ to $\psi = \psi$ alters the St-P. of the light through this strip by

$$\Delta(dI') = 0 \qquad \Delta(dM') = 0$$

$$\Delta(dC') = dC\,(1-\cos\psi) = dS\,\sin\psi \tag{46}$$

$$\Delta(dS') = +dC\,\sin\psi + dS\,(1-\cos\psi)$$

Whence the maximum change of intensity is, according to (21)

$$\Delta(\delta J) = \tfrac{1}{2}\left[(\Delta\delta C')^2 + (\Delta\delta S')^2\right]^{1/2}$$

$$= \sin\frac{\psi}{2}\,\sqrt{dC^2 + dS^2}$$

Again we note, since $dM^2 + dC^2 + dS^2 = dI^2$, that the maximum is reached when $dM = 0$. Thus the largest variation is

$$\Delta(\delta J) = dI\cdot\sin\psi/2 \tag{47}$$

and is independent of dC and dS. Consider the two cases $dC = 0$, $dS = dI$ and $dC = dI$, $dS = 0$. In the first case the

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light is circular, $\Delta(dC') = -dI \sin\psi$, $\Delta(dS') = dI(1-\cos\psi)$ and for small values of ψ the $\Delta(\delta Ip')$ vector is in the equatorial plane. According to the discussion in section (7) the analyzer must therefore be a Polaroid with the axis at 45° to x. In the second case we get for small values of ψ $\Delta(dC') = 0$, $\Delta(dS') = dI \sin\psi$ and to obtain maximum response a $\lambda/4$ plate must be inserted before the Polaroid of the receiver. Summarizing these results one may state that

a) to obtain the largest action of a narrow strip a $\lambda/4$ plate must be inserted between polarizer and analyzer, either before or after the shutter;

b) the maximum intensity variation is $\Delta I \sin\psi/2$, hence when ψ varies periodically the intensity variation is for small ψ of the same frequency as the supersonic;

c) the maximum variation is larger than that obtained for the conditions of max. efficiency of the uniform shutter. For crossed polaroids at 45° the narrow strip gives an intensity change of $\Delta(\delta J') = dI \sin^2 \psi/2$.

It becomes therefore apparent that it must be possible to construct an optical system for the shutter which creates a larger fluctuation output, and which at the same time avoids frequency doubling.

The foregoing results are taken into account in the operation of Kerr cells where a biasing field is introduced. This bias is chosen so as to create a $\psi = \pi/2$ and acts therefore in place of a $\lambda/4$ plate. From this analogy one might conclude that the application of a uniform stress in the block could improve the shutter. This, however, is not correct, because this stress would be equivalent to introducing a $\lambda/4$ plate before or after the uniform shutter, and we have already seen in section 11 that this procedure carries no advantage.

Frequency doubling and low response of the uniform shutter arise from the fact that, since photoelasticity is a linear effect, the change of ψ is equally often positive as it is negative within the area of the block. Hence in averaging over the whole are the integral over $\sin\psi/2$, where $\psi = \psi_m \sin 2\pi \frac{x}{\lambda_s}$, vanishes, and only the double frequency term, i.e. those in $\sin^2 \psi/2$ give a contribution.

To remedy this situation it is necessary to use a $\lambda/4$ plate in connection with the shutter. However, in order to prevent that the integration cancels the terms in $dI \sin\psi/2$ we must introduce a positive $\lambda/4$ plate ($\psi = r/2$) where ψ is positive and a negative $\lambda/4$ plate ($\psi = -\pi/2$) where ψ is negative or vice versa. This means of course that the $\lambda/4$ plate to be introduced must consist of strips of a width equal to $\lambda_s/2$, e.i. half the wavelength of the supersonic wave, or equal to the distance between successive nodes of the standing waves. In alternate strips the axis of the $\lambda/4$ plate must differ by 90°.

14) Theory of the "stripped" shutter.

Practical limitations arising from the fact that no $\lambda/4$ plate is accurate and cannot be accurate for a finite range of the spectrum, and from the circumstance that the strips may in the experiment not be accurately matched to the nodes, induce us to consider a somewhat more general problem. We assume that the incident natural light passes through a "stripped" polarizer before it enters the block. Each strip is $\lambda_s/2$ wide and consists of a Polaroid followed by a ψ plate, for which ψ is approximately $\pi/2$. According to the preceding discussion (M=0), the Polaroid must have its axis at 45 to x and the axis of the ψ plate is parallel to X. In successive strips the Polaroid axis or the axis of the ψ plate, or both, differ by 90°. We consider therefore 3 cases.

a) Stripped Polaroid, uniform ψ plate.

For this case the Stockes parameter of the light passing through the two sets of strips are:

First Set	Second Set
$I_1 = I$	$I_2 = I$
$M_1 = 0$	$M_2 = 0$
$C_1 = C$	$C_2 = -C$
$S_1 = S$	$S_2 = -S$

b) Uniform Polaroid, stripped ψ-plate.

$I_1 = I$	$I_2 = I$
$M_1 = 0$	$M_2 = 0$
$C_1 = C$	$C_2 = -C$
$S_1 = S$	$S_2 = S$

This case reverts to the uniform shutter when S = 0, and a and b become identical when C = 0.

c) Stripped Polaroid and stripped ψ-plate.

$I_1 = I$ $\qquad$ $I_2 = I$

$M_1 = 0$ $\qquad$ $M_2 = 0$

$C_1 = C$ $\qquad$ $C_2 = -C$

$S_1 = S$ $\qquad$ $S_2 = S$

Case (a) and (b) become idential when S = 0 and the case of the uniform shutter is obtained when C = 0.

The edges of the strips are assumed to deviate by a distance x_o from the position of the nodes. Thus if we take the position of the nodes to be at $x = 0$, $x = \lambda_s/2$, $x = \lambda_s$ etc., the strips are joined at the x positions $x = x_o$, $x = x_o + \lambda_s/2$, $x = x_o + \lambda_s$ etc. The integration over all strips is carried out in the same manner as in section 10 and gives for the Stockes parameter of the transmitted light

$$I' = I$$

$$M' = 0$$

$$C' = \frac{1}{\lambda_s}\left[C_1 \int_{x_o}^{x_o+\lambda_s/2} \cos\psi\, dx + C_2 \int_{x_o+\lambda_s/2}^{x_o+\lambda_s} \cos\psi\, dx\right] - \frac{1}{\lambda_s}\left[S_1 \int_{x_o}^{x_o+\lambda_s/2} \sin\psi\, dx + S_2 \int_{x_o+\lambda_s/2}^{x_o+\lambda_s} \sin\psi\, dx\right]$$

$$S' = \frac{1}{\lambda_s}\left[C_1 \int_{x_o}^{x_o+\lambda_s/2} \sin\psi\, dx + C_2 \int_{x_o+\lambda_s/2}^{x_o+\lambda_s} \sin\psi\, dx\right] + \frac{1}{\lambda_s}\left[S_1 \int_{x_o}^{x_o+\lambda_s/2} \cos\psi\, dx + S_2 \int_{x_o+\lambda_s/2}^{x_o+\lambda_s} \cos\psi\, dx\right]$$

where $\psi = \psi_m \sin 2\pi \frac{x}{\lambda_s}$. Substituting $z = \frac{2\pi x}{\lambda_s}$ and noting that

$$\frac{1}{2\pi}\int_{z_o}^{z_o+2\pi} \sin\ \psi\ dz = \int_0^{2\pi} \sin\ (\psi_m \sin z)\ dz = 0$$

$$\frac{1}{2\pi}\int_{z_o}^{z_o+2\pi} \cos\ \psi\ dz = \frac{1}{2\pi}\int_0^{2\pi} \cos\ (\psi_m \sin z)\ dz = J_o(\psi_m)$$

gives

$$C' = C_1 J_o(\psi_m) + (C_2-C_1)\frac{1}{2\pi}\int_{z_o+\pi}^{z_o+2\pi} \cos\ (\psi_m \sin z)\ dz$$

$$-(S_2-S_1)\frac{1}{2\pi}\int_{z_o+\pi}^{z_o+2\pi} \sin\ (\psi_m \sin z)\ dz$$

$$S' = (C_2-C_1)\frac{1}{2\pi}\int_{z_o+\pi}^{z_o+2\pi} \sin\ (\psi_m \sin z)\ dz + S_1 J_o(\psi_m)$$

$$+ (S_2-S_1)\frac{1}{2\pi}\int_{z_o+\pi}^{z_o+2\pi} \cos\ (\psi_m \sin z)\ dz$$

where

$$Z_o = \frac{2\pi X_o}{\lambda_s} \qquad (48)$$

Note that $\frac{1}{2\pi}\int_{z_o+\pi}^{z_o+2\pi} \cos\ (\psi_m \sin z)\ dz = \frac{1}{2} J_o(\psi_m)$. Hence with the abbreviations

$$K = J_o(\psi_m)$$

$$K_1 = -\frac{1}{2\pi}\int_{z_o+\pi}^{z_o+2\pi} \sin\ (\psi_m \sin z)\ dz = +\frac{1}{\pi}\int_{z_o}^{\pi/2} \sin(\psi_m \sin z)\ dz \qquad (49)$$

one arrives at the final result:

$$I' = I \qquad\qquad M' = 0$$

$$C' = \tfrac{1}{2}\,(C_1+C_2)K + (S_2-S_1)\,K_1 \qquad (50)$$

$$S' = (C_1-C_2)K_1 + \tfrac{1}{2}\,(S_1+S_2)\,K$$

Application to the three cases gives: $I' = I$, $M' = 0$, in all cases, but in Case a)

$$C' = 2SK_1 \qquad S' = 2CK_1 \qquad (51)$$

in Case b)

$$C' = CK - 2SK_1 \qquad S' = 0 \qquad (52)$$

in Case c)

$$C' = 0 \qquad S' = 2CK_1 + SK \qquad (53)$$

Noting that when the shutter is not vibrating $\psi_m = 0$, $K = J_0(\psi_m) = 1$, $K_1 = 0$ we arrive at the following table, which shows the change of the light character when the shutter is operated.

Table 1

Incident light	l		c		e	
Shutter	0	ψ_m	0	ψ_m	0	ψ_m
Transmitted for case a	n	pc	n	pc	n	pe
b	l	pl	n	pl	pl	pl
c	n	pc	c	pc	pc	pc

n = natural, l = linear, pl=partially linear, etc.

The largest possible intensity variation which can be produced with the stripped shutter is readily found to be

for Case (a) $\Delta J = K_1\, I$

(b) $\Delta J = C\tfrac{1}{2}(1-K) + SK_1$ (54)

(c) $\Delta J = S\tfrac{1}{2}(1-K) - CK_1$

To determine which of the three cases has greater advantages, we should note that $\frac{1}{2}(1-K)$ is the same term encountered in the uniform shutter. It gives rise to light suitable for audio reception and radio reception at the double frequency. K_1, however, being an odd function of γ_m gives a light modulation at the frequency of the supersonic and does not register in an audio amplifier since the average intensity is zero. The possibility of audio reception is not desirable, because it reduces the secrecy of transmission. Now in method (c) the term in (1-K) can be eliminated only by taking S=0, but this implies that the stripped phase plate is to be removed and this corresponds to a special case of (a). Thus, case (c), which also involves a more complicated construction, is eliminated. In case (b) the above secrecy feature is maintained provided the stripped phase plate is close to a $\lambda/4$ plate. Case (a), however, is decidedly the most favorable, because it eliminates every possibility of audio reception.

A second point favoring Case (a) follows from Table I. In case (b) the light transmitted by the shutter in operation is always partially plane polarized. Consequently any observer with a suitable Polaroid is able to detect that the transmission involves the polarization of light. In particular, if the ψ plate is not an exact $\lambda/4$ plate, the observer with a polaroid is able to "see" dot-dash communication on C.W. and with a photocell and audioamplifier he can intercept voice and MCW communication. In case (a), however, the transmitted light is usually partly elliptical. In the most simple case of all, i.e. when the polarizer is simply a "stripped" Polaroid, not followed by any ψ-plate, the transmission goes over partially circular light. The periodic fluctuation of ψ_m change the polarization periodically from right to left circular. This kind of light is practically indistinguishable from natural light. An observer with a good $\lambda/4$ plate and a Polaroid for the NAN light could only receive the signal with a Photocell-Radio receiver tuned to the frequency of the sender. It is absolutely impossible by any type of visual observation to notice intensity fluctuations or to find out that the transmission involves the polarization of light.

We conclude, therefore, that the best method is reached with case (a) when the stripped Polaroid is not followed by a $\lambda/4$ plate.

It may be noted from equation (54) that the term in K_1 can never be made larger than $K_1 I$, thus from this point of view method (a) is as good as any of the other methods. To close the discussion of the optical properties of the stripped shutter we have finally to consider the optics of the receiver.

With a stripped Polaroid S=0, C=I, the St-P. of the transmitted light are

$$I' = I, \quad M' = 0, \quad C' = 0, \quad S' = 2IK_1$$

A change from $\psi_m = 0$ to ψ_m changes the I_p vector from 0 to $\Delta I_p = S' = 2IK_1$. To transform the polarization change into an intensity change a $\lambda/4$ plate is necessary to turn the ΔI_p parallel to the equatorial plane. If this plate is not exact, i.e. if $\psi \neq \pi/2$, the component parallel to the equatorial plane is $2IK_1 \cos \psi$ and therefore the intensity variation

$$\Delta J = IK_1 \cos \psi \qquad (55)$$

We note that any reasonably good $\lambda/4$ plate with ψ differing less than 8° from $\pi/2$ over the spectral range is adequate because the deviation reduces the intensity fluctuation by less than 1% from the ideal case. Or in other words a $\lambda/4$ plate which is correct for 9000A can be used for a spectral range from 9000A to 10000A without introducing appreciable errors in our equation.

15) Frequency response characteristics of the stripped shutter.

The response of the stripped shutter is given by

$$J = \frac{I_t}{I_o} = K_1 = \frac{1}{\pi} \int_{z_o}^{\pi/2} \sin (\psi_m \sin z)\, dz \qquad (56)$$

When $z_o = 0$ we say the stripped Polaroid is "matched", i.e. the boundary between the strips coincides with the nodes of the supersonic. $z_o < \pi/2$ is thedegree of "unmatching". The evaluation of the function $K_1(\psi_m, z_o)$ is best carried out by means of Bessel functions.

Since

$$\sin (\psi_m \sin z) = 2 \sum_{o}^{\infty} J_{2n+1}(\psi_m) \sin (2n + 1) z$$

and $$\frac{2}{\pi} \int_{z_o}^{\pi/2} \sin (2n + 1) z\, dz = \frac{2}{\pi(2n+1)} \cos (2n+1) z_o$$

one finds

$$K_1(\psi_m z_o) = \frac{2}{\pi} \sum \frac{J_{2n+1}(\psi_m)}{(2n+1)} \cos (2n+1) z_o \qquad (57)$$

$$= \frac{2}{\pi} \left[J_1(\psi_m) \cos z_o + \frac{1}{3} J_3(\psi_m) \cos 3 z_o + \frac{1}{5} J \cos 5 z_o \right]$$

For the matched crystal the variation is largest, namely for $z_o = 0$

$$K_1(\psi_m 0) = \frac{2}{\pi} \sum \frac{J_{2n+1}(\psi_m)}{(2n+1)} \qquad (58)$$

For numerical calculations it is more convenient to use a power series of ψ_m:

$$K_1(\psi_m 0) = \frac{\psi_m}{\pi} \left[1 - \frac{\psi_m^2}{3^2} + \frac{\psi_m^4}{3^2 5^2} - \frac{\psi^6}{3^2 5^2 7^2} + \ldots\ldots \right] \qquad (59)$$

K_1 has a maximum at $\psi_m = 2.04$ where its value is $(K_1)_{max} = 0.395$. (See fig. 9)

C.W. Transmission

Of greater interest than $K_1(\psi_m 0)$ is the response of the vibrating shutter. This is found, for the matched position, from the Fourier analysis

$$K_1(\psi_m \sin \omega_s t) = \sum_{1}^{\infty} R_i \sin i \omega_s t$$

The largest term is R_1 which represents the amplitude of the signal as recorded by a radio receiver tuned to the frequency of the sender. Since

$$R_1 = \frac{1}{\pi} \int_0^{2\pi} K_1(\psi_m \sin x) \sin x \, dx$$

one finds from the power series (59)

$$R_1 = \frac{\psi_m}{\pi}\left[1 - \frac{\psi_m^2}{12} + \frac{\psi_m^4}{360} - \frac{\psi_m^6}{360.56} + \dots = \frac{2}{\pi}\sum_{n=0}^{\infty}\frac{\psi_m^{2n+1}(-1)^n}{(2n+2)!}\right. \qquad (60)$$

$R_1(\psi_m)$ has the first maximum $(R_1)_{max} = 0.463$ at $\psi_m = 2.33$.

K_1 and R_1 are plotted in fig. 9 and a discussion of the curve is given in the concluding section.

__MCW Transmission__

For an audio modulated shutter we replace ψ_m by $\psi_m(1+r \sin \omega_A t)$, where r is the percentage of modulation. The signal as registered by the R.F. receiver is given by $\left[R_1 + \sum A_i' r^i \sin i\,\omega_A t\right] \sin \omega_s t$. The modulation factor A_1', which has the same modulation frequency as the input is found from

$$A_1' = \frac{1}{\pi}\int_0^{2\pi} R_1\left[\psi_m(1+r \sin x)\right] \sin x \, dx$$

to

$$A_1' = \frac{\psi_m}{\pi}\left[1 - \psi_m^2\left(\frac{1}{8} + \frac{r^2}{16}\right) + \psi_m^4\left(\frac{1}{72} + \frac{r^2}{48} + \frac{r^4}{576}\right) \dots\right] \qquad (61)$$

This function is plotted in fig. 10 for $r \to 0$, i.e. when the modulation is small and for $r = 1$, i.e. for 100% modulation. It is noted that the maximum for A_1' occurs for all values of r at about $\psi_m = 1.25$. (The exact position is 1.228 for $r = 1$, 1.232 for $r = 3/4$, 1.240 for $r = 1/2$, 1.245 for $r = 1/4$ and 1.249 for $r \to 0$.) The height of the maximum decreases from 0.256 for $r \to 0$ to 0.230 for $r = 1$. Note that up to $\psi = 0.8$ ($\alpha = 12°$) M_1 differs very little from R_1. The difference $R_1 - A_1'$ gives the audio modulated intensity which vibrates at multiple frequencies $n\omega_A$. Up to $\psi = 1$, $\alpha = 14°$, the "distortion" $(R-M')/M'$ is less than 20%.

__Mechanical modulation__

The stripped shutter offers a new method of audio modulation of R.F. modulated light by moving the position x_o of the boundary between strips relative to the nodes of the supersonic. From equ. (57) we see that $K_1 = 0$ when $z_o = \frac{2\pi x_o}{\lambda_s} = \frac{\pi}{2}$, which occurs when the stripped Polaroid is "unmatched", with the boundaries halfway between nodes $(x_o = \frac{1}{2}\ \lambda_s/2)$. If $y_o = z_o t\ \pi/2$, where y_o corresponds to a displacement $x'_o = \frac{\lambda_s y_o}{2\pi}$ from the "unmatched" position we get:

$$K_1(\psi_m\ y_o) = \frac{2}{\pi} \sum_{m=1}^{\infty} \frac{J_{2n+1}(\psi_m)}{(2n+1)} (-1)^n \sin (2n+1)\ y_o \qquad (62)$$

or

$$K_1(\psi_m\ y) = \frac{\psi_m}{\pi} \sin y \left[1 - (\tfrac{\psi}{2})^2 (\tfrac{1}{2} + \tfrac{1}{18} \frac{\sin 3y_o}{\sin y_o} + (\tfrac{\psi_m}{2})^4 \right.$$

$$\left. (\tfrac{1}{12} + \tfrac{1}{72} \frac{\sin 3y_o}{\sin y_o} + \tfrac{1}{600} \frac{\sin 5y_o}{\sin y}) \right]$$

The radio signal changes therefore with the position of the shutter. This variation is given by

$$R_1(\psi_m\ y) = \frac{1}{\pi} \int_0^{2\pi} K_1(\psi_m \sin x,\ y) \sin x\ dx$$

and has the value

$$R_1(\psi_m\ y) = \frac{\psi_m}{\pi} \sin y_o \left[1 - (\tfrac{\psi_m}{2})^2 (\tfrac{3}{8} + \tfrac{3}{72} \frac{\sin^3 y_o}{\sin y}) \right. \qquad (63)$$

$$\left. + (\tfrac{\psi_m}{2})^5 \tfrac{15}{288} (1 + \tfrac{1}{6} \frac{\sin 3y_o}{\sin y_o} + \tfrac{1}{50} \frac{\sin 5y_o}{\sin y_o}) \ \cdots \right]$$

For $y_o = \pi/2$ this becomes identical with R_1. The dependence of $R_1(\psi_m, y_o)$ on y_o is very nearly a sine curve, except for increased values of ψ_m. For $\psi_m = 2$, which corresponds to nearly the largest value of R_1 the dependance $R_1(1,4)$ is given in Fig. 7. Even in this extreme case the deviations from $R_1(\psi_m\ y) = R_1(\psi_m) \sin y_o$ are small.

This result suggests that the audio modulation can be introduced by mechanical vibration of the stripped Polaroid. To this end $y_o = 0$ is chosen as rest position and the strips are vibrated with an amplitude $a < \pi/2$. The approximation $R_1(\psi_m\ y) = R_1(\psi_m) \sin y_o$ gives with $y_o = a \sin \omega_A t$

$$R_1(\psi_m \text{ a sin } \omega_A t) = R_1(\psi_m) \sin(a \sin \omega_A t)$$

Developing into a Fourier series

$$\sum V_i \sin i \; \omega_A t$$

furnishes, in analogy with the derivation of K_1 and R_1, for the amplitude of the fundamental

$$V_1 = R_1(\psi_m)\; R_1(a)$$

For small values of a $R_1(a) = \frac{a}{\pi}$ and if one expresses a in percentages of its maximum value $\pi/2$, i.e. $a = r\;\pi/2$ then $V_1 = R\;A_m'$ where A_m' has the same significance as A' in the previous sections and therefore

$$A_m' = \frac{1}{2} R_1(\psi_m) \qquad (64)$$

For larger values of r A_m' decreases. When r = 1 one finds, since $R_1(\pi/2) = 0.405$

$$A_m' = 0.4\; R_1(\psi_m)$$

The maximum of these functions occur at $\psi_m = 2.33$ ($\alpha = 31°$) and is 0.231 and 0.18, respectively. Due to the approximation we expect that actually the maxima are somewhat smaller and occur for somewhat lower α (about 25°).

The chief features of the stripped shutter may be summarized as follows:

a) The transmission possesses unusual secrecy features. It cannot be intercepted with audio receiver and the use of polarized light is not detectable.

b) C.W., M.C.W. and Voice can be received only with an R.F. receiver operated by a photocell and tuned to the frequency of the sender. A $\lambda/4$ plate must be inserted in front of the analyzer.

In contrast to the uniform shutter for which the intensity reaching the photocell is zero when the shutter is not operated, the average intensity passing through the stripped shutter is always the same. The difference between the types of intensity fluctuations for M.C.W. transmission of the two shutters is illustrated in Fig. 8.

16. Conclusions

To arrive at the physical interpretation of our theoretical results the following points should be kept in mind:

1) The theoretical quantities give the intensities or the amplitudes of the intensity fluctuations as fractions of the light intensity incident on the vibrating block, neglecting any subsequent intensity losses. It would be of greater practical value to express these quantities in terms of the light intensity which would reach the receiver when no instrument would be interposed, i.e. the intensity of the direct beam from the directed source. If this flux is denoted by J_o then the intensity reaching the block is about 0.35 J_o, because in the uniform and the stripped shutter at least 50% of the light is absorbed in the first Polaroid. Actually for most Polaroids (visible or NAN) the transmission through a single sheet is usually less than 40%. The "stripping" will introduce a certain light loss, but this can be neglected for the "matched" shutter because the light passing through the nodes is not acted upon. Only for the case of mechanical modulation (a reduction of perhaps 10%) is due to the stripping. Loss due to reflection at normal incidence on each surface of the block and the $\lambda/4$ plate may be assumed to amount to 5%, thus the correction factor for the light reaching the analyzing Polaroid is about 0.31 for the uniform and 0.30 for the stripped shutter. Finally, the analyzer is not ideal. Instead of transmitting 50% of natural light it transmits only 35%, which gives an additional correction factor of 0.7. Thus, when referred to the intensity of the source, all our values have to be reduced by a factor of about 0.2. The scale on the right side of figures 8 and 9 refers to fraction of J_o and is only approximate.

2) The A quantities give the amplitude of the audio modulated component of the light, referred to 100% modulation. Now experimentally it has been found that the actual modulation attained in the block is smaller even if the driving oscillator is 100% modulated. The modulation decreases with increasing frequency from about 80% at 100 cycles to 20% at 2000 cycles.

Choosing an average value of 50% for the entire audio range we should introduce a correction factor of 0.1 when the intensities are referred to J_0 and to the practical conditions.

In the case of mechanical modulation the same factor is applicable, because a vibration of full amplitude $\lambda_s/4$ will introduce considerable distortion of the audio signal. It is estimated that the distortion is not serious when r = 0.5, i.e. when the amplitude is $\lambda_s/8$. In this connection it should be mentioned that in the present shutters λ_s is about 0.5 cm. The curves refer therefore to an amplitude of vibration of the stripped Polaroid sheet of only 0.06 cm, which appears to make the method a practical possibility.

3) In discussing the practical advantages of the several methods, the difficulties in producing large values of ψ_m or α is the deciding factor. Using visible light and blocks of 1" thickness it is found that in blocks with an α larger than 30° excessive heating of the block is unavoidable. Since it gives rise to spurious birefringence, this should be avoided. Thus for the above conditions $\psi_m = 2.25$ appears to be the upper limit. Now $\psi_m = \frac{2\pi t}{\lambda_o}(n_1-n_2)$, where λ_o is the optical wavelength, t the thickness of the block. The birefringence $(n_1 - n_2)$ can be assumed to be independent of λ_o. It follows that for a 1" block the upper limit of ψ_m for NAN is reduced to 1.5, or $\alpha = 21°$. To be sure this value can be raised by choosing a thicker block, but this will probably increase the heating, and it seems reasonable to conclude that $\alpha = 30°$ is the upper practical limit for NAN transmission, and that for continuous operation $\alpha < 20°$ would be desirable.

On the basis of these considerations and the curves in figs. 9 and 10, the choice between the different procedures is quite clear. For C.W. transmission the stripped shutter possesses all advantages over the uniform shutter. Satisfactory operation is reached for $\alpha = 20°$. An increase of α from 20° to 30° does not increase the output sufficiently to make it worth while. The uniform shutter has in this range an over three times smaller output than the stripped shutter, and even an increase of α to 40° would not make it superior to the stripped shutter at $\alpha = 20°$.

For MCW or Voice transmission we have four cases to consider:

a) RF reception with stripped shutter, given by A_1'.

b) Audio reception with uniform shutter, given by A_0'.

SECRET

c) RF reception with uniform shutter, given by A'_2.

d) RF reception with mechanical modulation, given by A'_M.

The two curves labeled 0 and 1 for A' and A'_M refer to small percentage and 100% modulation, respectively. The curve to be used is in between. A'_o and A'_2 are valid for small modulation, but are only slightly reduced, in the important range, when 50% modulation is used.

Again it is quite clear that for $\alpha < 20°$ the stripped shutter with electrical modulation is superior. Mechanical modulation can not reach the same output, even if the maximum vibration amplitude is used. The uniform shutter, however, becomes superior when an $\alpha > 24°$ can be applied. It is at present doubtful whether the difficulties encountered with strong vibrations can be overcome. It would certainly not be efficient to use RF reception with the uniform shutter, because a gain of only about 25% over the stripped shutter could be expected, and to attain it the α would have to be nearly doubled. In other words the driving energy, which is proportional to ψ_m^2 would have to be more than 4 times larger.

The same reasons apply also, though to a lesser degree for audio reception. The main disadvantage of audio reception arises from the fact that it records any other audio frequency changes of the light level, arising from atmospheric changes, and that it can readily be jammed by any audio modulated source.

Summarizing, it is concluded that the stripped shutter is preferable for CW and audio reception. It operates satisfactorily for values of α within reach of the experiments and the transmission has all the secrecy features lacking in the other methods.

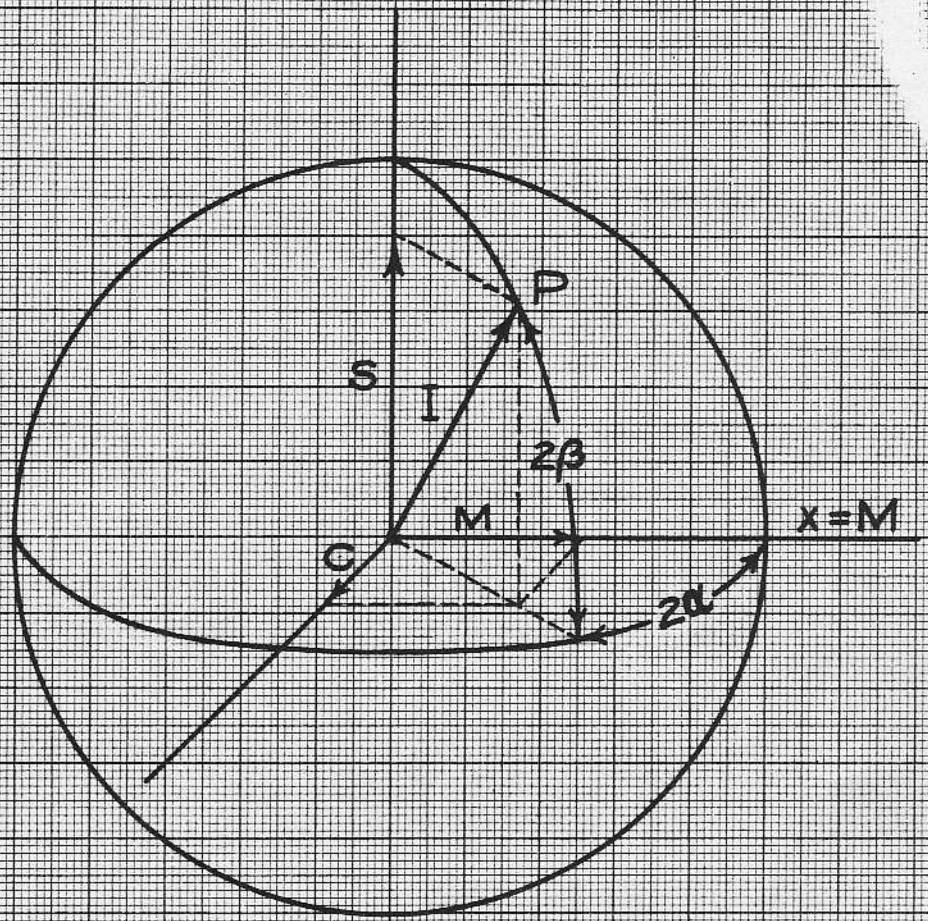
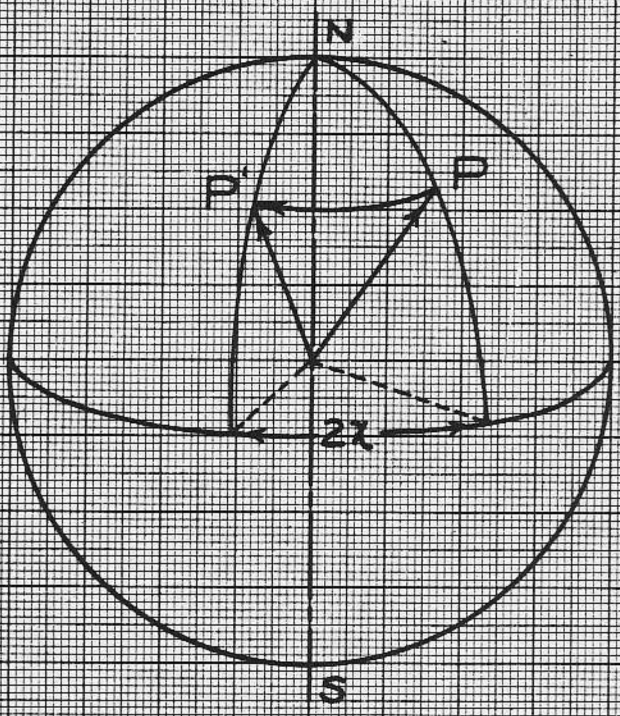
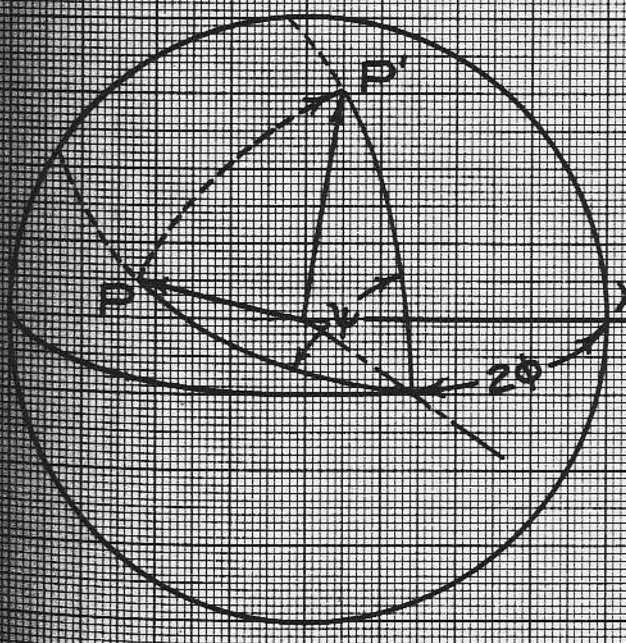


FIG. 1

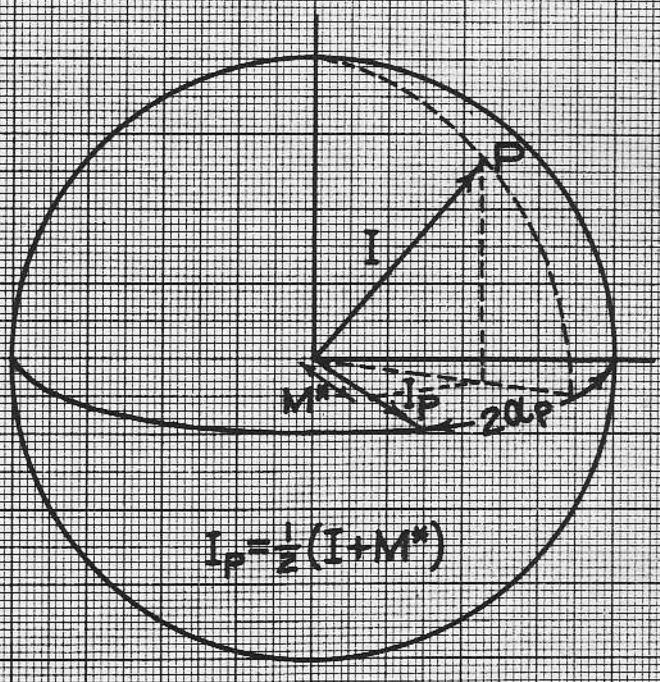
POINCARÉ SPHERE AND STOKE'S PARAMETERS: M,C,S



OPTICAL ACTIVITY



BIREFRINGENCE



POLARIZER

FIG. 2

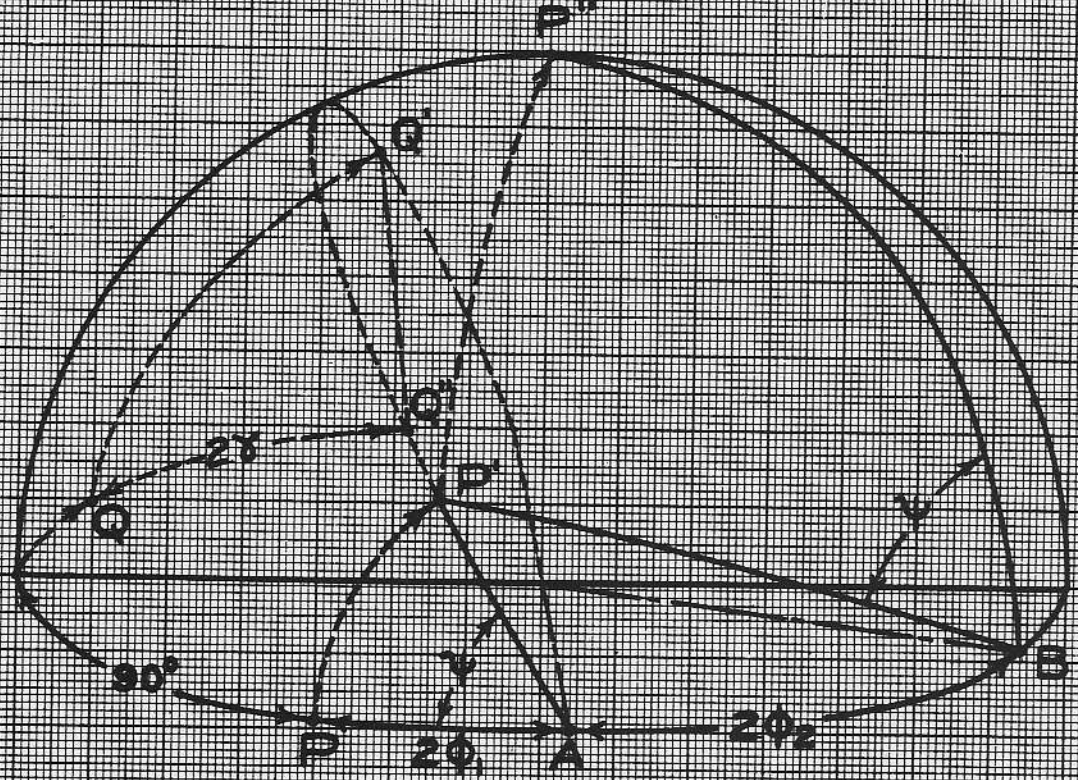


FIG. 3

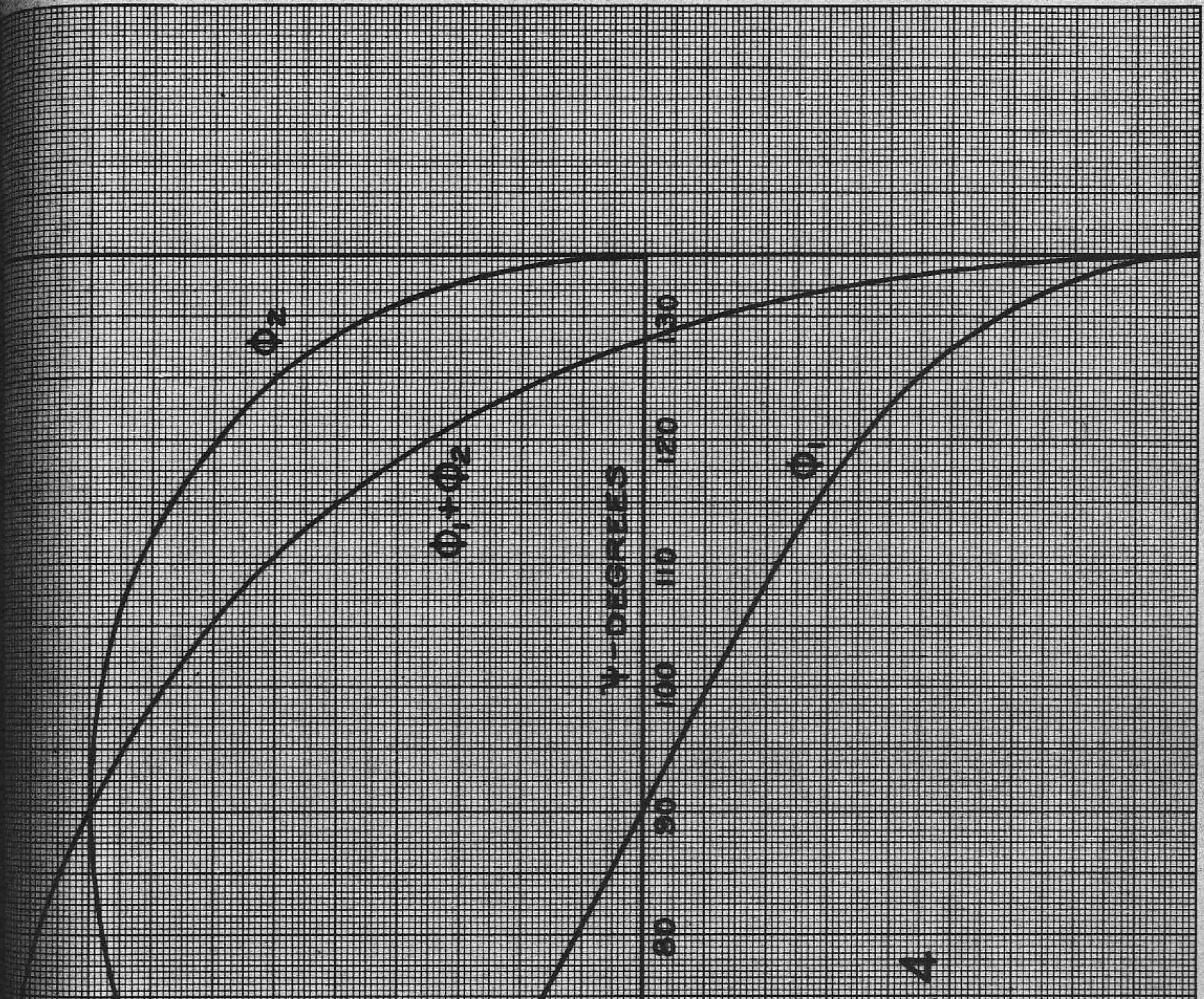

Ψ-DEGREES
50
60
70
80
90
100
110
120
130
Φ-DEGREES
50
40
30
20
10
0
-10
-20
-30
-40
Φ1
Φ2
Φ1+Φ2

FIG. 4

SECRET

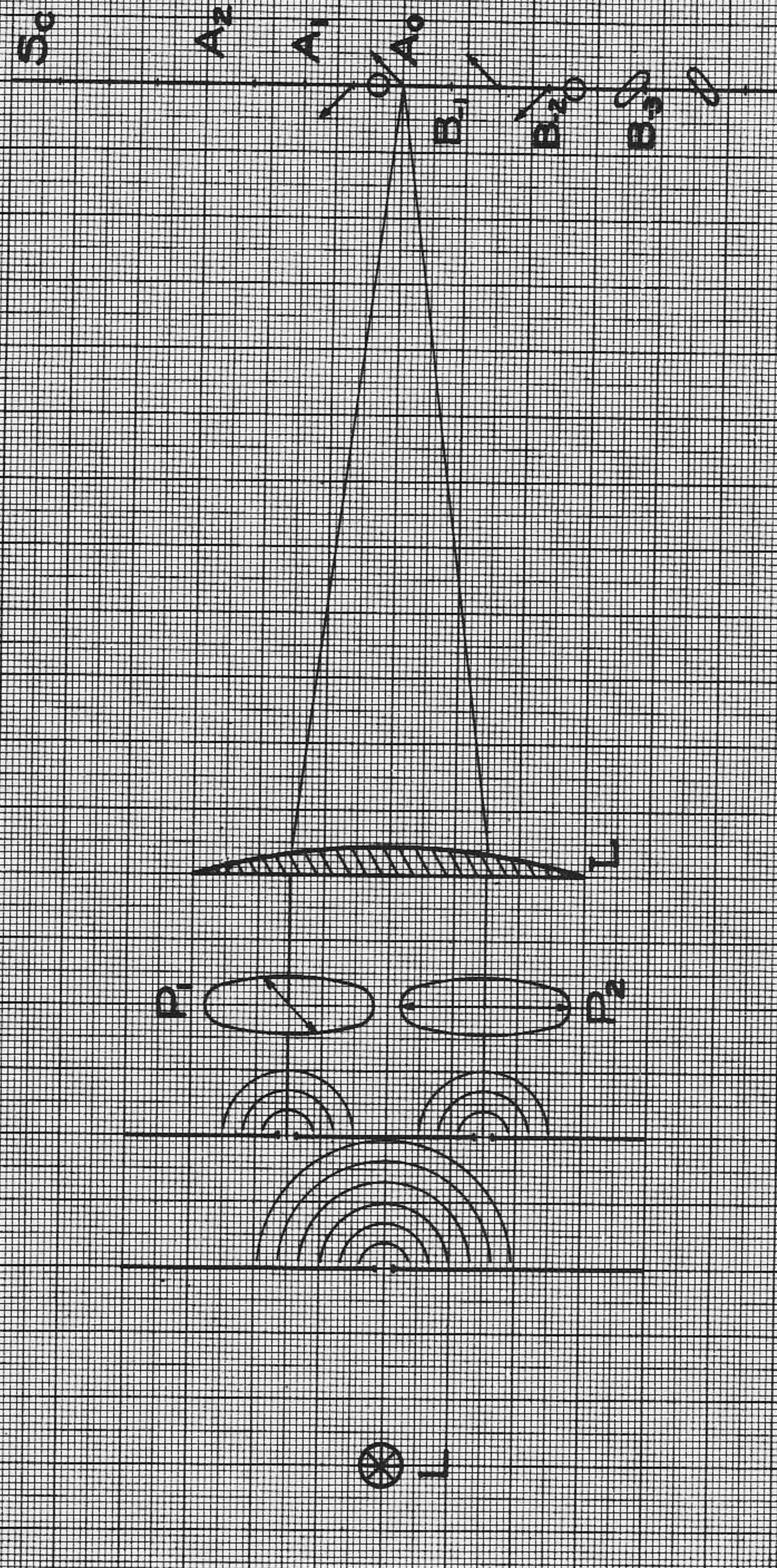
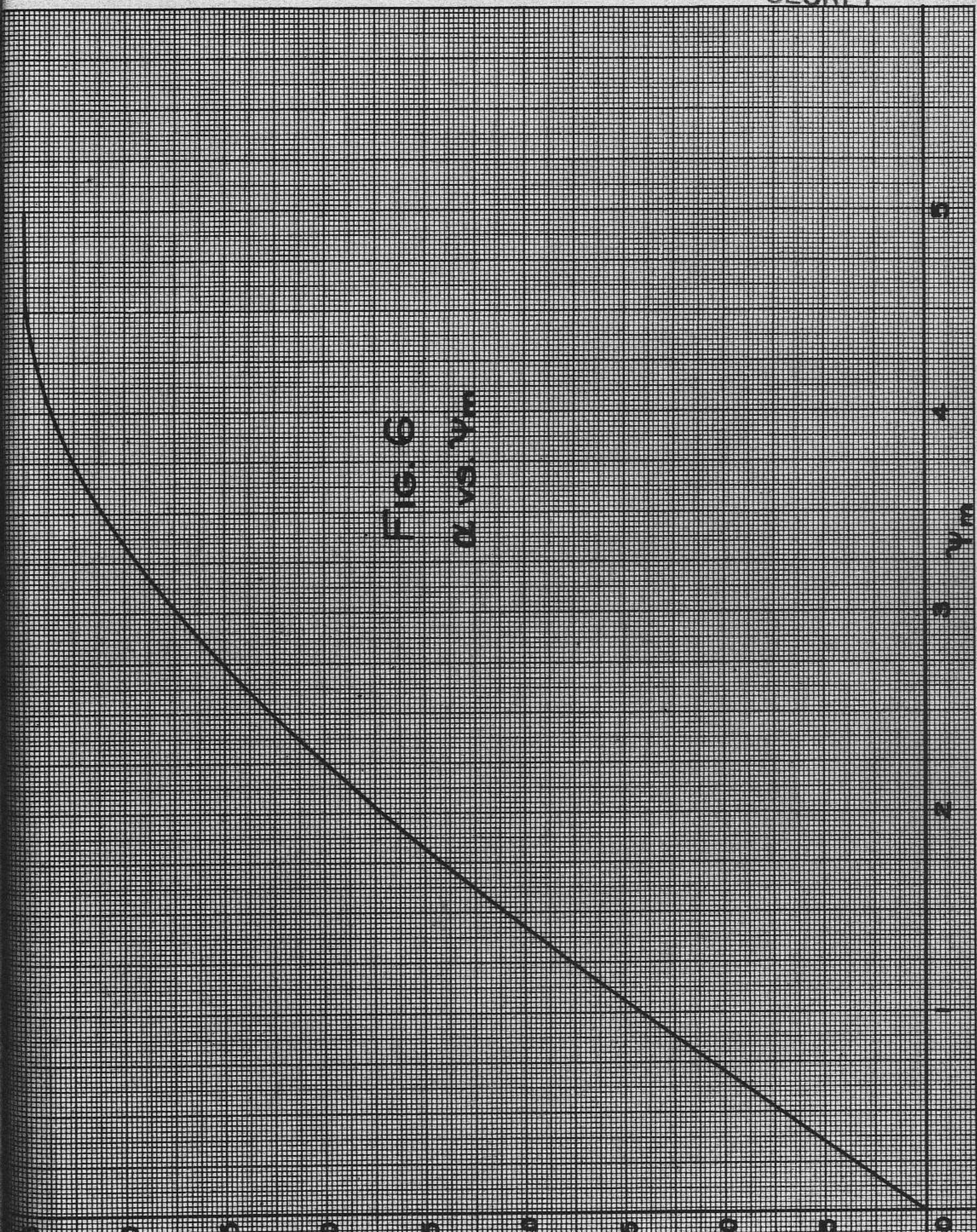


FIG. 5

SECRET



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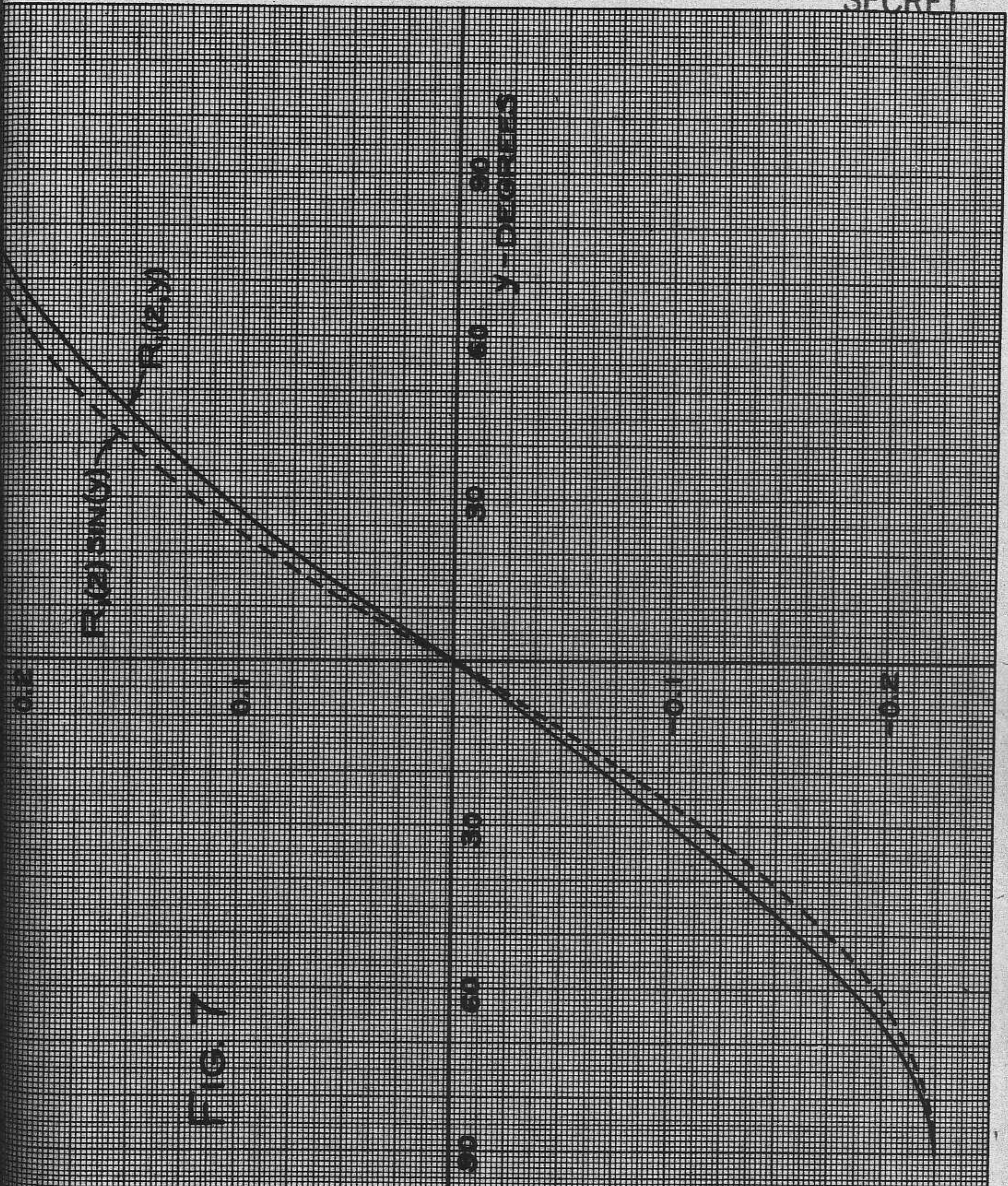


FIG. 7

SECRET

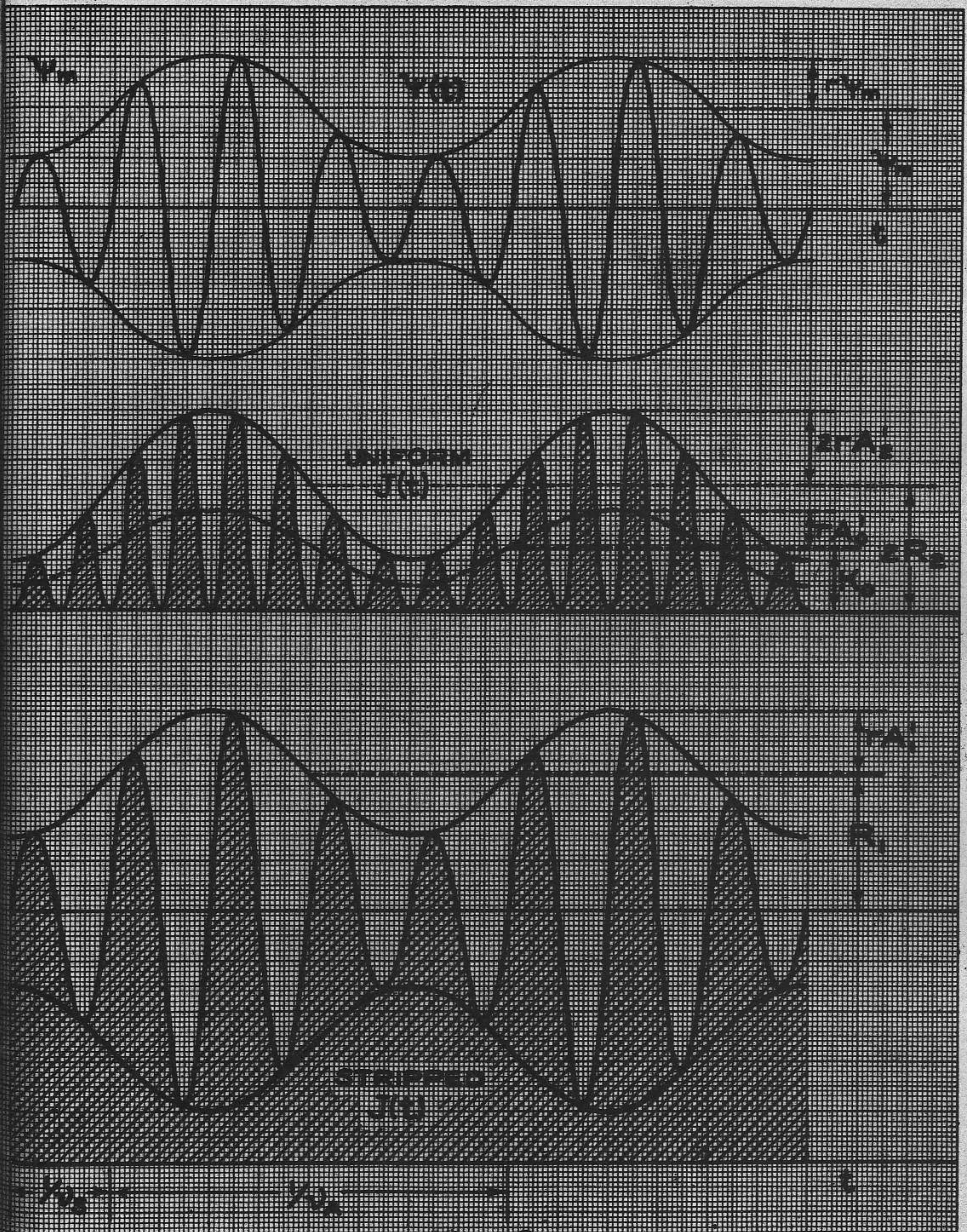
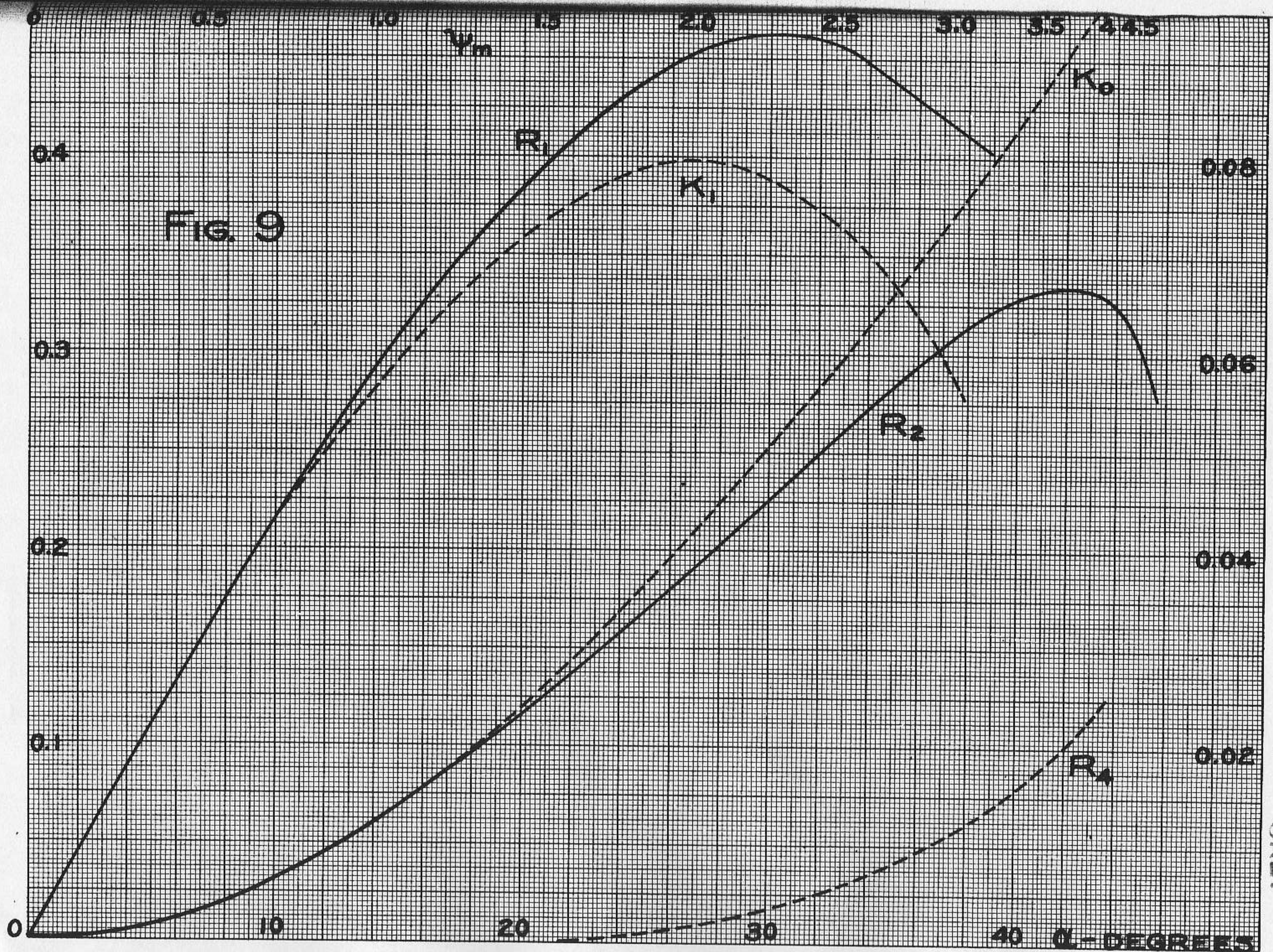


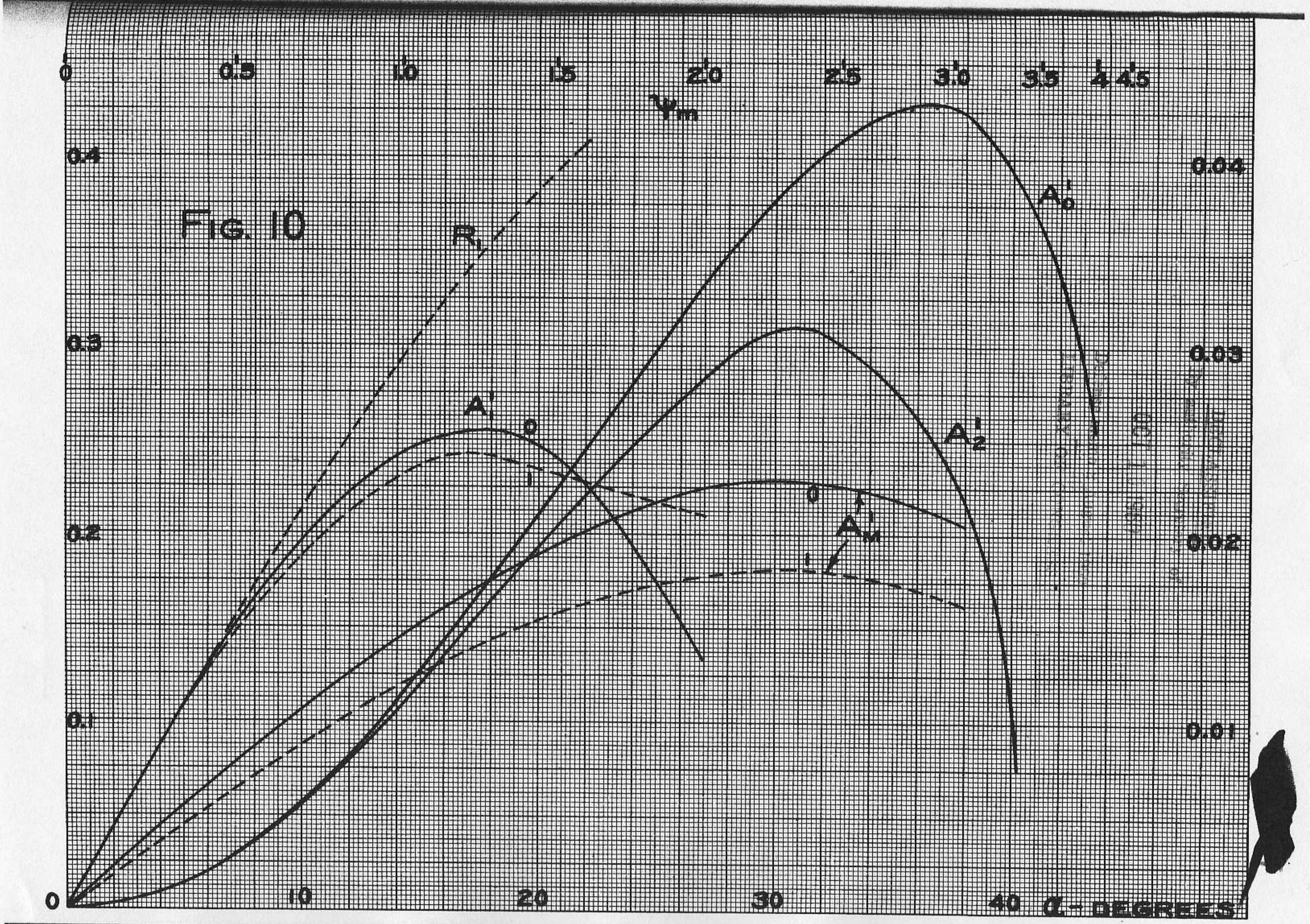
FIG. 8

SECRET



0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4
4.5
ψ_m
0.4
0.3
0.2
0.1
0
0.08
0.06
0.04
0.02
10
20
30
40
α - DEGREES
R_1
K_1
K_0
R_2
R_4

FIG. 9



0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4
4.5
ψ_m
0.4
0.3
0.2
0.1
0.04
0.03
0.02
0.01
R_1
A_1'
A_0'
A_2'
A_M'
0
1
10
20
30
40
α - DEGREES

FIG. 10