V. *On the Reflexion of Polarized Light from Polished Surfaces, Transparent and Metallic.*

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Introduction.

AMONG the experimenters who have made the reflexion of polarized light the object of their researches, there is no one to whom science is more indebted than to M. JAMIN, whose accurate observations are a model for subsequent observers. His first paper on this subject was published (1847) in the 19th volume of the 'Annales de Chimie et de Physique,' 3rd series, p. 296, on Metallic Reflexion.

In this remarkable paper M. JAMIN verified many of the previous observations of BREWSTER, and added many of his own. He employed two distinct methods in these experiments,—

1st. *The method of Comparative Intensities*—by observing the relative intensities of the same beam of light reflected from a polished surface, composed partly of glass and partly of the substance to be examined.

2nd. *The method of Multiple Reflexions*, previously known from the researches of BREWSTER.

The optical constants used by JAMIN in this paper are—

(a) The angle (i_1) of maximum polarization.

(b) The angle (A) whose tangent is the ratio of I to J, the square roots of the intensities reflected in the plane of incidence, and in the perpendicular plane.

(c) The coefficient (ε) used by CAUCHY, which is connected with the other two constants by means of theoretical equations.

By the first method of observation, M. JAMIN determines the constants i_1 and ε for the following substances:—

1. Steel,
2. Speculum metal;

and by the second method of observation, he determines i_1 and A for

3. Silver,

and i_1 for

4. Zinc,

and gives the details of experiments on

5. Copper,

from which the optical constants may be found.

M. JAMIN's next paper on Metallic Reflexion appeared in 1848, in the Annales de Chim. et de Phys. 3rd series, vol. xxii. p. 311. In this paper he makes use of the second method of observation, by multiple reflexion, and gives valuable tables of the results of his experiments with the various colours of the spectrum on the seven following metallic substances:—

1. Steel.
2. Speculum metal.
3. Silver.
4. Zinc.
5. Copper.
6. Brass.
7. Bell metal.

From these Tables the constants i_1 and A may be inferred.

In 1850 M. JAMIN published his well known paper "On the Reflexion of Light at the Surface of Transparent Bodies," in the Ann. de Chim. et de Phys. 3rd series, vol. xxix. p. 263. In this series of experiments he used a new method of observation, founded on the Quartz Compensator of BABINET. In this elaborate and important paper he publishes the details of his experiments on the following substances:—

1. Fire Opal,
2. Hyalite,
3. Realgar,
4. Blende,
5. Diamond,
6. Fluor-spar,
7, 8. Two kinds of glass,

and, in addition, gives in a Table at the end of the paper the constants of many other transparent bodies.

M. JAMIN has also published, in 1851, in the Ann. de Chim. et de Phys. vol. xxxi. p. 165, a memoir "On the Reflexion of Light at the Surface of Liquids," in which he determines the optical constants of many liquids.

It occurred to me that the method of observation employed by JAMIN for transparent bodies might be advantageously used in the case of metals; and I was thus led to commence the series of experiments the results of which are recorded in the following pages.

In these experiments I have added many metallic substances to JAMIN's list, and have re-examined the metals observed by him by a different method.

In transparent bodies I have examined a few not experimented on by JAMIN, and investigated in detail the form of the reflected ellipse, under varying conditions of incidence and azimuth.

In the course of my paper I have employed for the second optical constant one more readily determined than those usually employed, but which is readily deduced from the constants A and k of JAMIN.

At the close of the paper I shall give a Table containing a comparison of the constants found by JAMIN and myself for all the bodies which we have both examined.

Some years ago, in making observations on polarized light, I found that by adjusting properly the azimuth of the incident polarized beam, and allowing it to fall at the angle of principal incidence, I could obtain a reflected beam of circularly polarized light.

On repeating the experiment with different polished surfaces, I found that the *coefficient of reflexion*, or whatever property it is that gives a surface a metallic reflexion, might be conveniently expressed by the cotangent of the azimuth at which an incident beam of plane-polarized light should be placed so as to give, on reflexion at the principal incidence, a reflected beam of circularly polarized light.

The following paper contains an account of my experiments on many substances, and a Table of their Coefficients of Reflexion and Refraction, determined with as much accuracy as I was able to attain with the instruments at my disposal.

The apparatus used by me consisted of a large graduated circle (horizontal), provided with two moveable arms, each furnished with graduated circles (vertical); and the large horizontal circle was capable of being hung vertically, so as to allow of experiments being made on liquids as well as solids. The substance to be examined was placed on a stage provided with adjusting screws, so as to bring the surface exactly into the centre, or intersection of the axes of the polarizing and analysing arms. These arms were mounted with Nicol prisms, made for me by DUBOSCQ of Paris, and without sensible deviation. The light employed was generally sunlight, but I sometimes used a moderator lamp with colza oil.

I employed the quartz compensator described by M. JAMIN*, for the purpose of converting the elliptically polarized reflected light into plane-polarized light, before allowing it to pass through the analyser.

The instrument used by me in making my observations on the reflexion of polarized light, was made by Mr. GRUBB of Dublin for the late Professor M^cCULLAGH, and was presented to me, shortly after M^cCULLAGH's death, by his brother. It is substantially the same as that described by M. JAMIN in vol. xxix. Ann. de Chim. et de Phys. sér. 3. I procured from M. DUBOSCQ SOLEIL, of Paris, a compensator of JAMIN's pattern, and had it adapted to my own apparatus.

In making my observations I used the following precautions:—

1. The zero of both polarizer and analyser was determined by direct observation with red sunlight, reflected at the angle of polarization of several glasses found to give a reflected beam capable of being completely cut off by the Nicol prism.

2. The Nicol prisms themselves were carefully tested and found to have no deviation.

3. Each of my recorded observations is the mean of four or five; and when these differed from each other by more than 20′, I took the precaution of repeating them

* Annales de Chimie et de Physique, sér. 3. vol. xxix. p. 263 *et seq.*

again, on another day, with my eye fresh and unfatigued, before I finally adopted my mean.

4. I frequently repeated the observations, with the incident light polarized at an equal angle, at the opposite side of the plane of incidence; and also reversing the polarizer and analyser, so as to read the opposite sides of their scales.

The following definitions will explain the sense in which I use certain terms.

The *Azimuth* of a beam of plane-polarized light is the angle which its plane of polarization makes with the plane of incidence.

The *Index of Refraction* is the ratio which the sine of the angle of incidence bears to the sine of the angle of refraction.

The *Coefficient of Refraction* is the tangent of the *Principal Incidence.*

The *Principal Incidence* is that angle of incidence at which rays polarized in any azimuth have the major axis of the reflected elliptic light in the plane of incidence; or at which the components of the reflected beam, in and perpendicular to the plane of incidence, differ by 90° in phase.

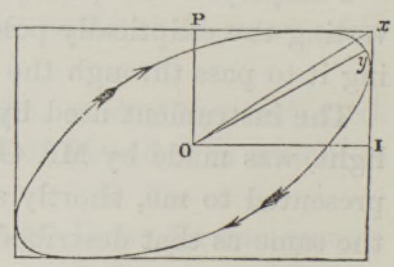
This angle is nearly the same as BREWSTER'S Angle of Polarization or Maximum Polarization.

The *Coefficient of Reflexion* is the Cotangent of the Azimuth of an incident beam of plane-polarized light, which after reflexion at the principal incidence becomes circularly polarized.

The *Principal Components* of the incident and reflected light are the components in and perpendicular to the plane of incidence.

The following preliminary investigation will serve to show the principles on which I have tabulated the results of my experiments:—

Let the elliptically polarized reflected beam be represented, as in the annexed figure, inscribed in a rectangle, whose sides are parallel to O I and O P, the plane of incidence and perpendicular plane.



Let O x be the diagonal of the circumscribed rectangle, and O y the axis of the ellipse; it is required, from the difference of phase of the light in the planes O I and O P, and knowing the direction of the line O x, to find the direction of O y and the ratio of the axes of the ellipse.

The angle x O I $=\alpha'$ is the azimuth of the reflected beam, measured by the analyser, after it has lost its elliptic polarization in the compensator; and the difference of phase of O I and O P is measured in the compensator itself, by the displacement necessary to reduce the elliptically-polarized to plane-polarized light.

We may imagine, to aid our conception, but without hypothesis, that a material point traverses the ellipse, and that its coordinates are

$$\xi = \text{A} \sin(kt+e),$$
$$\eta = \text{B} \sin(kt+e'),$$

where $e'-e$ is the difference of phase between the beams O I and O P, and A, B are the lines O I and O P.

Eliminating t, we find

$$\frac{\xi^2}{A^2}+\frac{\eta^2}{B^2}-2\cos(e'-e)\frac{\xi\eta}{AB}=\sin^2(e'-e). \quad \ldots \ldots \quad (1.)$$

In this ellipse, the angle φ made by the axis with the plane of incidence is found from the well-known expression

$$\tan 2\varphi=\frac{2E}{D-F},$$

belonging to the ellipse

$$Dx^2+2Exy+Fy^2=\text{const.}$$

Substituting for D, E, F their values from (1.), we find

$$\tan 2\varphi=\tan 2\alpha'\cos(e'-e); \quad \ldots \ldots \quad (2.)$$

φ denoting the angle y O I, and α' the angle x O I. But if a and b denote the axes of the ellipse, it can be proved that

$$\frac{b^2}{a^2}=\frac{(D+F)+(D-F)\sec 2\varphi}{(D+F)-(D-F)\sec 2\varphi};$$

or substituting from (1.) and (2.),

$$\left.\begin{aligned}\frac{a}{b}&=\sqrt{-\cot(\varphi+\alpha')\cot(\varphi-\alpha')}\\ \frac{b}{a}&=\sqrt{-\tan(\varphi+\alpha')\tan(\varphi-\alpha')}\end{aligned}\right\} \ldots \ldots \quad (3.)$$

From equations (2.) and (3.), I calculate the position of the elliptic axes and their ratio.

The angle α' is obtained by direct measurement with the analysing prism; and $e'-e$ may be found, as follows, from the compensator.

In the compensator made for me by M. DUBOSCQ, I find that 39·43 represents the zero, *i. e.* the position in which the compensator affects equally the light in and perpendicular to the plane of incidence; the number of graduations corresponding to a difference of half a wave (180°) I found to be

Red lamplight (colza oil)	15·43
Red sunlight	15·37
White lamplight (colza oil) . . .	13·29

If, therefore, C denote the reading of the compensator in any experiment, the difference of phase of the two principal beams will be expressed for red sunlight, in degrees, by the expression

$$(C-39{\cdot}43)\times\frac{180^\circ}{15{\cdot}37},$$

and by a corresponding formula for the other kinds of light.

The angle thus measured by the compensator is not the difference of phase between the principal components of the reflected light until it is increased by 180°, because experiment shows that in the act of reflexion there is this constant difference between the two components, in addition to the varying difference of phase, depending on incidence, azimuth, and nature of polished surface. We therefore use the formula

$$e'-e=180^\circ+\left(C-39\cdot43\right)\times\frac{180^\circ}{i}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.)$$

where i denotes the interval corresponding to 180° for the light used.

In tabulating my experiments, I give the original measurements of the analyser and compensator, and use the equations (2.), (3.), and (4.) to calculate the other columns.

I. Munich Glass (a).

The first experiments I shall record were made with glass procured from Munich by the late Professor McCullagh. I have four rhombs made of it, whose index of refraction I determined by the following experiments:—

Table I.—Munich Glass (a).

Rhomb.	Angle.	Minimum deviation* of red light.	Refractive index.
No. 1.	44° 56′ 0″	31° 43′ 30″	1·6229
No. 2.	54 28 30	41 28 0	1·623 0
No. 3.	39 50 0	27 17 0	1·6227
No. 4.	59 58 30	48 22 0	1·6221
		Mean	1·6227

Calculated Angle of Polarization =58° 21′.

I also found the refractive indices of No. 2 for the extreme red and violet rays to be 1·6190 and 1·6555, which indicates a dispersive power in the glass of 0·0573.

This glass was found to contain the following constituents:—

Silica	42·25
Oxide of Lead . . .	46·35
Lime	0·45
Alkalies (by diff.) . .	10·95
	100·00

The following Tables contain my observations on this glass:—

* In all my experiments the red light used was passed through the same piece of red glass, which was very homogeneous.

TABLE II.—Munich Glass (a). (September 20, 1854.)
Azimuth of Polarizer=20°. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	φ.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
34° 30′	39·69	11° 35′	2° 41′	+11° 34′	89·66	29° 23′
52 30	42·55	2 5	36 30	+ 1 40	45·80	5 42
53 30	44·09	2 0	54 31	+ 1 10	35·25	5 29
54 30	45·81	2 0	74 38	+ 0 32	29·71	5 29
55 30	48·68	2 6	108 13	− 0 39	28·68	5 45
56 30	49·86	2 37	122 2	− 1 24	25·89	7 9
73 30	53·88	11 45	169 4	−11 34	26·93	29 45

The principal incidence is therefore 54° 57′.

The last column of this Table is thus found:—

Let the principal components of the incident polarized beam, in and perpendicular to the plane of incidence, be $\cos\alpha$ and $\sin\alpha$ (unity denoting the incident beam); and let I and J denote what a unit of light becomes after reflexion, in and perpendicular to the plane of incidence respectively; then the principal components of the reflected beam are $I\cos\alpha$ and $J\sin\alpha$, and therefore

$$\frac{J}{I}=\tan\alpha'\cot\alpha; \qquad (5.)$$

and the angle $\tan^{-1}\left(\frac{J}{I}\right)$ may be found from this equation without any difficulty.

According to the theory of FRESNEL,

$$\frac{J}{I}=\frac{\cos(i+r)}{\cos(i-r)},$$

an expression which vanishes at the polarizing angle ($i+r=90°$), and therefore $\tan^{-1}\left(\frac{J}{I}\right)$ ought at this angle of incidence to vanish also; but we find, not only in this experiment, but in those which follow, that it does not vanish, but only reaches a minimum, the tangent of which is sensibly equal to what I have called the Coefficient of Reflexion*.

In fact, let λ denote the angle whose cotangent is this coefficient. Then $I\cos\lambda$, $J\sin\lambda$ are the principal components of the reflected light, which by definition is circularly polarized, and therefore $I\cos\lambda=J\sin\lambda$, and

$$\cot\lambda=\frac{J}{I}.$$

The coefficients of Refraction and Reflexion, as determined by this experiment, are therefore

Coefficient of Refraction $=\tan 54°\ 57'=1·4255$.
Coefficient of Reflexion $=\cot 84°\ 31'=0·0960$.

* Strictly speaking the angle of incidence at which the maximum is reached is found to be *somewhat* less than the Principal Incidence.

TABLE III.—Munich Glass (a). (June 26, 1854.)

Azimuth of Polarizer =45°. White lamplight (Colza oil).

Incidence.	Compensator.	Analyser.	$\epsilon'-\epsilon-180°$.	φ.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
43° 37′	39·54	18° 30′	3° 11′	+18° 28′	47·79	18° 30′
48 37	40·07	10 55	10 17	+10 45	29·42	10 55
50 45	40·61	8 10	17 36	+ 7 48	23·37	8 10
51 45	41·11	6 45	24 22	+ 6 10	20·70	6 45
52 45	42·67	6 10	45 29	+ 4 21	13·03	6 10
53 52	43·15	5 22	50 21	+ 3 27	13·86	5 22
54 20	44·46	5 1	69 44	+ 1 45	12·15	5 1
55 20	46·30	5 36	94 38	− 0 27	10·23	5 36
56 20	48·18	6 15	120 6	− 3 10	10·58	6 15
57 40	50·07	7 35	145 41	− 6 19	13·52	7 35
58 40	51·00	9 39	158 16	− 9 0	16·16	9 39
60 35	51·60	11 10	166 24	−10 53	22·34	11 10
65 40	51·98	18 11	171 33	−18 2	22·84	18 11
75 35	52·50	30 25	178 40	−30 25	∞	30 25

The principal incidence is therefore 55° 8′, and the minimum value of $\tan^{-1}\left(\frac{J}{I}\right)$ is 5° 1′, or Circular limit=84° 59′. Therefore the

Coefficient of Refraction=1·4352.
Coefficient of Reflexion =0·0877.

TABLE IV.—Munich Glass (a). (July 28, 1854.)

Azimuth of Polarizer =80°. Red Sunlight.

Incidence.	Compensator.	Analyser.	$\epsilon'-\epsilon-180°$.	φ.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
34° 30′	39·72	74° 0′	3° 23′	+74° 1′	88·20	31° 35′
43 30	40·13	61 45	8 11	+61 53	16·90	18 10
48 30	40·76	49 15	15 34	+49 24	7·62	11 34
50 30	42·15	38 0	31 49	+36 49	3·62	7 51
51 30	42·80	33 30	39 26	+30 36	3·09	6 40
52 30	43·85	28 0	51 42	+21 17	2·70	5 22
53 30	45·41	25 45	69 57	+11 39	2·28	4 52
54 30	47·20	26 34	90 54	− 0 36	1·99	5 2
55 30	48·80	28 45	109 37	−13 53	2·02	5 31
56 30	50·30	34 0	127 10	−28 7	2·26	6 47
57 30	51·20	40 0	137 42	−38 18	2·64	8 25
58 30	51·21	42 30	137 49	−41 38	2·82	9 10
60 30	52·14	53 45	148 42	−55 8	3·76	13 31
65 30	52·77	66 30	156 4	−67 47	6·59	22 5
70 45	53·18	75 15	160 52	−75 56	12·30	33 49

Principal Incidence =54° 27′. $\tan^{-1}\left(\frac{J}{I}\right)$=4° 57′, or Circular limit =85° 3′.

Coeff. of Refraction =1·3993. Coeff. of Reflexion =0·0866.

Table V.—Munich Glass (a). (August 7, 1854.)

Azimuth of Polarizer $=85°$. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	ϕ.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
34 30	39·68	80 12	2 55	+80 13	98·30	26 52
52 30	43·37	44 30	46 5	+44 17	2·37	4 55
53 30	44·33	39 54	57 19	+35 47	1·86	4 11
54 30	46·12	38 24	78 16	+20 28	1·36	3 58
55 30	48·13	41 50	101 47	−30 44	1·26	4 28
56 30	49·45	44 0	117 13	−42 49	1·64	4 50
57 30	50·42	51 0	128 34	−54 24	2·15	6 10
73 30	53·62	81 0	166 1	−81 15	26·76	28 55

Principal Incidence $=54°\ 59'$. $\tan^{-1}\left(\frac{J}{I}\right)=3°\ 58'$, or Circular limit $=86°\ 2'$.

Coeff. of Refraction $=1·4272$. Coeff. of Reflexion $=0·0693$.

Table VI.—Munich Glass (a). (September 27, 1854.)

Azimuth of Polarizer $=85°\ 45'$. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	ϕ.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
54 30	46·05	43 26	77 23	+37 56	1·25	4 2
54 45	46·75	43 20	85 38	+26 18	1·09	4 1
55 0	46·90	43 8	87 24	+17 24	1·08	3 59
55 15	47·53	43 15	94 46	−26 49	1·11	4 0
55 30	48·05	45 30	100 51	−47 39	1·21	4 20

Principal Incidence $=55°\ 6'$. $\tan^{-1}\left(\frac{J}{I}\right)=3°\ 59'$, or Circular limit $=86°\ 1'$.

Coeff. of Refraction $=1·4334$. Coeff. of Reflexion $=0·0696$.

Table VII.—Munich Glass (a). (September 27, 1854.)

Azimuth of Polarizer $=85°\ 55'$. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	ϕ.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
54 30	45·96	45 30	76 24	+47 7	1·27	4 10
54 45	46·65	45 12	84 28	+47 4	1·10	4 7
55 0	47·00	45 5	88 34	+48 19	1·02	4 6
55 15	47·72	45 40	96 59	−50 25	1·13	4 11
55 30	48·00	46 30	100 16	−53 12	1·20	4 18

Principal Incidence $=55°\ 7'$. $\tan^{-1}\left(\frac{J}{I}\right)=4°\ 6'$, or Circular limit $=85°\ 54'$.

Coeff. of Refraction $=1·4343$. Coeff. of Reflexion $=0·0717$.

TABLE VIII.—Munich Glass (*a*). (September 26, 1854.)
Azimuth of Polarizer =86°. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	ϕ.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
34 30	39·77	83 20	3 58	+83 21	120·50	30 53
52 30	43·03	53 30	42 7	+56 16	2·73	5 24
53 30	44·67	47 0	61 18	+49 8	1·70	4 17
54 0	45·25	46 20	68 5	+48 34	1·48	4 11
54 30	46·13	46 11	78 23	+50 48	1·23	4 10
54 45	46·48	46 0	82 28	+52 28	1·14	4 8
55 0	46·91	45 45	87 30	+60 29	1·06	4 6
55 15	47·33	47 0	92 25	−74 27	1·08	4 17
55 30	48·07	48 20	101 5	−60 39	1·25	4 30
56 0	48·86	50 15	111 30	−58 25	1·53	4 48
56 30	49·40	51 30	116 38	−58 37	1·71	5 2
57 30	50·20	57 0	126 0	−63 34	2·26	6 9
73 30	53·69	83 15	166 50	−83 25	37·96	30 35

Principal Incidence =55° 8′. $\tan^{-1}\left(\frac{J}{I}\right)$=4° 6′, or Circular limit =85° 54′.

Coeff. of Refraction =1·4352. Coeff. of Reflexion =0·0717.

TABLE IX.—Munich Glass (*a*). (September 21, 1854.)
Azimuth of Polarizer =87°. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	ϕ.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
34 30	39·48	84 50	0 27	+84 50	∞	30 6
52 30	42·87	60 0	40 14	+63 33	3·27	5 11
53 30	44·06	55 0	54 10	+60 56	2·16	4 17
54 0	44·93	54 20	64 21	+63 59	1·78	4 11
54 30	45·82	54 0	74 45	+70 30	1·52	4 8
54 45	46·34	53 54	80 50	+76 48	1·43	4 7
55 0	46·60	53 55	83 53	+80 50	1·40	4 7
55 15	47·23	53 34	91 15	−87 59	1·35	4 4
55 30	47·92	55 30	99 20	−78 33	1·48	4 22
56 0	48·40	56 30	104 57	−74 21	1·64	4 32
56 30	49·08	59 30	112 54	−72 28	1·97	5 5
57 30	50·00	64 30	123 40	−72 48	2·72	6 16
73 30	53·34	85 0	165 5	−85 10	44·50	30 55

Principal Incidence =55° 13′. $\tan^{-1}\left(\frac{J}{I}\right)$=4° 4′, or Circular limit =85° 56′.

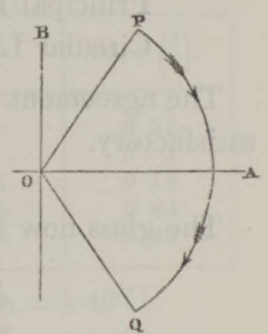
Coeff. of Refraction =1·4397. Coeff. of Reflexion =0·0711.

Collecting together the preceding results, and denoting by λ the azimuth of the plane of polarization of the incident light, which on reflexion at the principal incidence will produce, on reflexion, circularly polarized light, and calling it the Circular Limit, we obtain

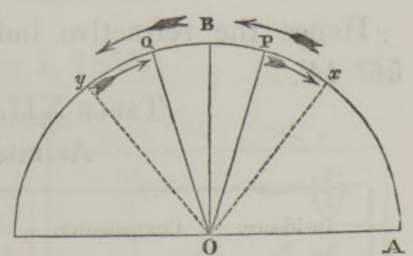
TABLE X.—Constants of Munich Glass (a).

Azimuth of Polarizer.	Principal Incidence.	Circular Limit.	Coefficient of Refraction.	Coefficient of Reflexion.
20° 0′	54° 57′	84° 31′	1·4255	0·0960
45 0	55 8	84 59	1·4352	0·0877
80 0	54 27	85 3	1·3993	0·0866
85 0	54 59	86 2	1·4272	0·0693
85 45	55 6	86 1	1·4334	0·0696
85 55	55 7	85 54	1·4343	0·0717
86 0	55 8	85 54	1·4352	0·0717
87 0	55 13	85 56	1·4397	0·0711
Means........	55° 0′ 37″	85° 32′ 30″	1·4287	0·0780

The movement of the axis of the reflected ellipse differs according as the azimuth of the incident light is less or greater than the circular limit. This is shown in Plate VIII. fig. A, on which the values of φ are laid down for different angles of incidence in the two cases in which the azimuth of the incident light is 80° and 87°.



When the azimuth of the incident light is less than λ, the circular limit, the axis of the ellipse moves as in the annexed figure. Let P O A be the azimuth of the incident light, and Q O A equal to P O A; P O is the position of the axis major corresponding to 0° incidence; O A is the position of the axis major in the plane of incidence, corresponding to the principal incidence; and O Q is the position of the axis corresponding to 90° incidence.



When, however, the azimuth of the incident light is greater than the circular limit, the axis major moves from P to x, as in the annexed figure, then back from x to y, passing through B at the principal incidence, and finally from y to Q. Let P O A be the azimuth of the incident light, and Q O B equal to P O B. At the incidence 0°, O P is the position of the axis major; as the incidence increases from 0° to the principal incidence, the axis major moves from O P to O x and turns back, attaining the position O B at the principal incidence; and as the incident angle increases from the principal incidence to 90°, the axis major moves from O B to O y, and back again to O Q.

Having ascertained the truth of the preceding laws of the movement of the axis major of the elliptically polarized light, I made the following experiments. Having removed the compensator, I set the polarizer at 88° and 89°, and found that the analyser gave a minimum of light at 90°, showing that the axis major of the ellipse was perpendicular to the plane of incidence.

All the experiments already given were made with the Munich glass (*a*) No. 1. I made the following experiment with (*a*) No. 2, in order to establish fully the identity of the pieces of glass with regard to reflexion, as they are certainly identical in their refractive indices.

TABLE XI.—Munich Glass (*a*). (October 11, 1854.)
Azimuth of Polarizer =86°. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	φ.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
34° 30′	39·70	83° 20′	3° 8′	+83° 20′	∞	30° 53′
52 30	43·33	53 30	45 37	+56 48	2·53	5 24
53 30	44·70	50 0	61 39	+55 11	1·73	4 46
54 0	45·60	49 0	72 11	+57 20	1·41	4 36
54 30	46·30	47 30	80 22	+58 48	1·21	4 22
54 45	46·79	46 45	85 58	+65 30	1·10	4 15
55 0	47·31	46 0	92 11	−66 15	1·05	4 9
55 15	47·70	47 30	96 45	−63 20	1·16	4 22
55 30	48·00	48 30	100 16	−62 17	1·24	4 31
56 0	48·83	50 30	109 58	−59 50	1·50	4 51
56 30	49·10	52 30	113 8	−62 9	1·64	5 12
57 30	49·95	57 0	123 5	−64 32	2·15	6 9
73 30	53·40	82 30	163 27	−82 48	26·98	27 59

Principal Incidence =54° 53′. Coeff. of Refraction =1·4220.
Circular Limit =85° 51′. Coeff. of Reflexion =0·0725.

The agreement of these values with those given for No. (1) in Table X. is sufficiently satisfactory.

II. MUNICH GLASS (*b*).

The glass now to be described is a rhomb which gave me the following values:—

Angle of rhomb . . . . =54° 30′
Minimum deviation of standard red . . . 34 2

Hence the refractive index of this red is 1·5244, and the angle of polarization 56° 44′.

TABLE XII.—Munich Glass (*b*). (October 10, 1854.)
Azimuth of Polarizer =45°. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	φ.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
54° 15′	39·43	1° 0′	0° 0′	+1° 0′	∞	1° 0′
54 30	39·43	0 43	0 0	+0 43	∞	0 43
54 45	39·43	0 30	0 0	+0 30	∞	0 30
55 0	39·43	0 10	0 0	+0 10	∞	0 10
55 15	54·13	2 15	172 0	−2 14	209·4	2 15

Principal Incidence =55° 1′. Coeff. of Refraction =1·4290.
Circular Limit =89° 50′. Coeff. of Reflexion =0·0029.

TABLE XIII.—Munich Glass (*b*). (September 29, 1854.)

Azimuth of Polarizer =80°. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	ϕ.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
34° 30′	39·43	72° 30′	0° 0′	+72° 30′	∞	29° 13′
52 30	39·57	20 15	1 38	+20 15	∞	3 38
53 30	39·65	11 30	2 34	+11 29	127·3	2 3
53 45	39·65	10 15	2 34	+10 14	135·6	1 49
54 0	39·65	7 30	2 34	+ 7 29	160·2	1 20
54 15	39·65	6 0	2 34	+ 5 59	179·9	1 4
54 30	39·73	1 20	3 30	+ 1 20	171·8	0 13
54 45	54·25	4 15	173 23	− 4 13	114·9	0 45
55 0	54·25	6 0	173 23	− 5 57	80·6	1 4
55 15	54·25	8 30	173 23	− 8 27	54·8	1 30
55 30	54·35	9 37	174 33	− 9 34	62·8	1 43
56 30	54·35	17 30	174 33	−17 26	37·5	3 11
57 30	54·36	25 25	174 41	−25 21	28·3	4 47
58 30	54·45	31 0	175 44	−30 58	30·3	6 3
73 30	54·68	70 0	178 25	−70 0	90·5	25 51
83 30	54·68	76 30	178 25	−76 30	164·3	36 18

Principal Incidence =54° 35′. Coeff. of Refraction =1·4063.

Circular Limit =89° 47′. Coeff. of Reflexion =0·0037.

TABLE XIV.—Munich Glass (*b*). (October 10, 1854.)

Azimuth of Polarizer =87°. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	ϕ.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
54° 15′	39·43	15° 40′	0° 0′	+15° 40′	∞	0° 51′
54 30	39·43	2 15	0 0	+ 2 15	∞	0 7
54 45	54·13	3 45	172 0	− 3 43	114·5	0 12
55 0	54·13	7 40	172 0	− 7 36	56·1	0 24
55 15	54·13	14 20	172 0	−14 13	30·0	0 46

Principal Incidence =54° 36′. Coeff. of Refraction =1·4071.

Circular Limit =89° 53′. Coeff. of Reflexion =0·0020.

TABLE XV.—Munich Glass (*b*). (October 1, 1855.)

Azimuth of Polarizer =88°. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	ϕ.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
33° 37′	39·43	86° 45′	0° 0′	+86° 45′	∞	31° 36′
43 37	39·43	83 45	0 0	+83 45	∞	17 41
53 37	39·43	37 30	0 0	+37 30	∞	1 32
54 37	39·43	359 45	180 0	− 0 15	∞	0 1
55 37	39·43	322 0	180 0	−38 0	∞	1 34
56 37	39·43	301 30	180 0	−58 30	∞	3 16
63 37	39·43	278 0	180 0	−82 0	∞	13 57
73 37	39·43	274 30	180 0	−85 30	∞	23 55

Principal Incidence =54° 37′. Coeff. of Refraction =1·4080.

Circular Limit =89° 59′. Coeff. of Reflexion =0·0001.

TABLE XVI.—Munich Glass (b). (October 5, 1855.)

Azimuth of Polarizer =89°. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	ϕ.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
33° 37′	39·43	88° 0′	0° 0′	+88° 0′	∞	26° 34′
43 37	39·43	85 25	0 0	+85 25	∞	12 17
53 37	39·43	55 0	0 0	+55 0	∞	1 26
54 37	39·43	358 0	180 0	− 2 0	∞	0 2
55 37	39·43	307 30	180 0	−52 30	∞	1 2
56 37	39·43	292 0	180 0	−68 0	∞	2 28
63 37	39·43	275 0	180 0	−85 0	∞	11 17
73 37	39·43	272 15	180 0	−87 45	∞	23 57

Principal Incidence =54° 35′. Coeff. of Refraction =1·4063.

Circular Limit = 0° 2′. Coeff. of Reflexion =0·0006.

From this and the four preceding Tables the following results may be collected.

TABLE XVII.—Constants of Munich Glass (b).

Azimuth of Polarizer.	Principal Incidence.	Circular Limit.	Coefficient of Refraction.	Coefficient of Reflexion.
45°	55° 1′	89° 50′	1·4290	0·0029
80	54 35	89 47	1·4063	0·0037
87	54 36	89 53	1·4071	0·0020
88	54 37	89 59	1·4080	0·0001
89	54 35	89 58	1·4063	0·0006
Means.........	54° 40′ 48″	89° 53′ 24″	1·4113	0·0019

III. PARIS GLASS.

This glass was supplied to me by M. DUBOSCQ of Paris. Its refractive constants were found to be as follows:—

Angle of prism	59° 55′
Minimum deviation of extreme red	37 35
Minimum deviation of extreme violet	39 13
Index of refraction of extreme red	=1·5059
Index of refraction of extreme violet	=1·5246

TABLE XVIII.—Paris Glass. (October 1, 1855.)

Azimuth of Polarizer $=88°$. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	φ.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
33° 37′	39·43	86° 20′	0° 0′	+86° 20′	∞	28° 35′
43 37	39·43	84 30	0 0	+84 30	∞	19 56
53 37	39·43	63 45	0 0	+63 45	∞	4 3
54 37	40·19	50 20	8 53	+50 24	12·76	2 25
55 37	42·43	25 30	35 6	+22 39	4·24	0 57
55 52	44·44	19 30	58 36	+11 26	3·41	0 43
56 7	46·75	17 30	85 38	+ 1 32	3·18	0 38
56 22	48·39	18 45	104 41	− 5 30	3·07	0 41
56 37	50·22	27 30	126 14	−20 5	2·65	1 2
57 37	52·70	48 0	155 15	−48 18	4·59	2 13
58 37	53·27	60 0	161 55	−60 38	7·32	3 28
63 37	54·00	80 0	170 28	−80 7	36·85	11 12
73 37	54·40	87 0	175 8	−87 1	180·90	33 41

Principal Incidence $=56°\ 10'$. Coeff. of Refraction $=1·4919$.
Circular Limit $=89°\ 22'$. Coeff. of Reflexion $=0·0110$.

The compensator was then set at 47·12, which corresponds with a difference of phase of 90°, between the principal components of the reflected light; and the compensator being thus set, the angle of incidence was determined by trial, for which the dark band was centrally placed. The incidence so found is the principal incidence. Having thus found the principal incidence, I changed the azimuth of the polarizer, and read the analyser, obtaining the following results.

TABLE XIX.—Paris Glass. (October 1, 1855.)

Compensator $=47·12=90°$. Red Sunlight.

Polarizer.	Analyser.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
89° 30′	48° 0′	1·110	0° 33′
89 0	32 0	1·600	0 37
88 0	18 0	3·077	0 39
87 0	13 20	4·219	0 43
86 0	10 0	5·671	0 43
85 0	9 0	6·314	0 48
80 0	4 30	12·706	0 48
70 0	2 45	20·819	1 0
60 0	1 40	34·367	0 58
50 0	1 10	49·103	0 59
40 0	0 54	63·656	1 4
30 0	0 47	73·139	1 21
20 0	0 37	92·908	1 42
10 0	0 24	143·237	2 16

Principal Incidence $=56°\ 7'$. Coeff. of Refraction $=1·4891$.
Circular Limit $=89°\ 24'$. Coeff. of Reflexion $=0·0104$.

The last column of this Table shows that the value of $\left(\frac{J}{I}\right)$ increases slightly as the azimuth of the polarizer diminishes.

Combining the preceding results, we find

Table XX.—Constants of Paris Glass.

No.	Principal Incidence.	Circular Limit.	Coefficient of Refraction.	Coefficient of Reflexion.
XVIII.	56° 10′	89° 22′	1·4919	0·0110
XIX.	56 7	89 24	1·4891	0·0104
Means..........	56° 8′ 30″	89° 23′	1·4905	0·9107

IV. Fluor-Spar.

The specimen of fluor-spar on which I made my experiments was transparent and blue. The following are the results I obtained.

Table XXI.—Fluor-Spar. (September 11, 1855.)

Azimuth of Polarizer =80°. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	ϕ.	$\frac{a}{b}$	$\tan^{-1}\left(\frac{J}{I}\right)$.
33° 37′	39·43	73° 30′	0° 0′	+73° 30′	∞	30° 46′
43 37	39·43	60 45	0 0	+60 45	∞	17 29
53 37	39·43	10 30	0 0	+10 30	∞	1 53
54 37	39·43	0 30	0 0	+ 0 30	∞	0 5
55 37	39·43	351 45	180 0	− 8 15	∞	1 28
58 37	39·43	327 0	180 0	−33 0	∞	6 32
63 37	39·43	305 0	180 0	−55 0	∞	14 8
73 37	39·43	288 15	180 0	−71 45	∞	28 8

Principal Incidence =54° 40′. Coeff. of Refraction =1·4106.
Circular Limit =89° 55′. Coeff. of Reflexion =0·0014.

Table XXII.—Fluor-Spar. (September 20, 1855.)

Azimuth of Polarizer = 88°. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	ϕ.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
53° 37′	39·43	43° 30′	0° 0′	+43° 30′	∞	1° 54′
54 7	39·43	31 0	0 0	+31 0	∞	1 12
54 37	39·43	15 0	0 0	+15 0	∞	0 32
55 7	39·43	344 30	180 0	−15 30	∞	0 33
55 37	39·43	331 0	180 0	−29 0	∞	1 7

Principal Incidence = 54° 52′. Coeff. of Refraction = 1·4211.
Circular Limit = 89° 28′. Coeff. of Reflexion = 0·0093.

From the preceding results combined, we obtain the following constants of fluor-spar.

Table XXIII.—Constants of Fluor-Spar.

No.	Principal. Incidence.	Circular Limit.	Coefficient of Refraction.	Coefficient of Reflexion.
XXI.	54° 40′	89° 55′	1·4106	0·0014
XXII.	54 52	89 28	1·4211	0·0093
Means............	54 46	89° 41′ 30″	1·4158	0·0053

V. Glass of Antimony.

The specimen of this glass with which I experimented was given to me by Professor Apjohn.

Table XXIV.—Glass of Antimony. (October 5, 1855.)

Azimuth of Polarizer = 80°. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	ϕ.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
33° 37′	39·43	75° 0′	0° 0′	+75° 0′	∞	33° 21′
43 37	39·43	66 20	0 0	+66 20	∞	21 55
53 37	39·82	38 50	4 33	+38 49	27·43	8 5
55 37	40·16	25 10	8 32	+25 1	17·84	4 44
57 37	41·90	10 30	28 54	+ 8 50	9·89	1 52
58 7	43·14	8 30	43 24	+ 6 16	9·86	1 31
58 37	46·19	6 20	79 5	+ 1 13	9·18	1 7
59 7	49·50	7 30	117 48	− 3 34	8·62	1 20
59 37	51·18	10 15	137 22	− 7 41	8·30	1 50
61 37	53·21	26 30	161 13	−25 46	7·77	5 2
63 37	53·70	39 30	166 57	−39 21	13·92	8 16
73 37	54·25	68 20	173 33	−63 25	27·04	23 56

Principal Incidence = 58° 44′. Coeff. of Refraction = 1·6468.
Circular Limit = 88° 53′. Coeff. of Reflexion = 0·0195.

TABLE XXV.—Glass of Antimony. (October 5, 1855.)

Azimuth of Polarizer =89°. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	φ.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
33° 37′	39·43	89° 0′	0° 0′	+89° 0′	∞	45° 0′
43 37	39·43	87 45	0 0	+87 45	∞	23 57
53 37	39·65	83 20	2 34	+83 20	∞	8 30
55 37	39·91	80 0	5 36	+80 3	56·19	5 39
57 37	41·25	64 30	21 17	+65 30	6·93	2 6
58 7	42·43	55 40	35 6	+57 46	3·44	1 28
58 37	45·72	48 0	73 35	+55 12	1·36	1 7
58 47	47·00	47 0	88 34	+72 11	1·09	1 4
59 7	48·76	49 30	109 9	−67 26	1·24	1 10
59 37	50·35	60 0	127 45	−60 23	9·36	1 44
61 37	52·80	78 30	156 26	−79 22	12·75	6 9
63 37	53·68	82 15	166 43	−82 27	32·36	7 19
73 37	54·10	88 45	171 38	−88 46	281·50	38 40

Principal Incidence =58° 50′. Coeff. of Refraction =1·6533.

Circular Limit =88° 56′. Coeff. of Reflexion =0·0186.

TABLE XXVI.—Glass of Antimony. (October 5, 1855.)

Compensator =47·12=90°. Red Sunlight.

Polarizer.	Analyser.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
89°	50° 0′	1·192	1° 11′
88	28 45	1·823	1 6
87	21 15	2·571	1 10
85	12 40	4·449	1 9
80	6 25	8·892	1 8
70	3 10	18·075	1 9
50	1 35	36·177	1 20
30	0 52	66·105	1 30
10	0 37	92·908	3 29
			Mean = 1° 28′ 0″

Principal Incidence =58° 52′. Coeff. of Refraction =1·6555.

Circular Limit =88° 46′. Coeff. of Reflexion =0·0215.

It is to be remarked, that in Table XXV., in which the azimuth of the polarizer is greater than the circular limit, the movement of the axis of the ellipse follows the same law as that of the Munich glass already described.

From the foregoing Tables, the optical constants of Glass of Antimony may be thus inferred:—

TABLE XXVII.—Constants of Glass of Antimony.

No.	Principal Incidence.	Circular Limit.	Coefficient of Refraction.	Coefficient of Reflexion.
XXIV.	58° 44′	88° 53′	1·6468	0·0195
XXV.	58 50	88 56	1·6533	0·0186
XXVI.	58 52	88 46	1·6555	0·0215
Means	58° 48′ 40″	88° 51′ 40″	1·6519	0·0199

VI. QUARTZ (*a*). *Natural surface. Plane of incidence perpendicular to optical axis.*

TABLE XXVIII.—Quartz (*a*). (October 13, 1855.)

Azimuth of Polarizer $=88°$. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	ϕ.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
33° 37′	39·43	87° 10′	0° 0′	+87° 10′	∞	35° 12′
43 37	39·43	84 20	0 0	+84 20	∞	19 23
53 37	39·90	67 45	3 9	+67 46	59·16	4 52
55 37	41·02	42 0	18 36	+41 50	6·09	1 48
56 7	43·40	32 0	46 27	+27 21	2·70	1 15
56 37	46·82	24 30	86 27	+ 4 4	2·20	0 55
57 7	50·53	35 0	129 52	−30 12	2·34	1 16
57 37	51·28	48 0	138 38	−49 0	2·65	2 13
58 37	52·10	63 15	148 14	−65 31	4·50	3 58
63 37	53·30	80 45	162 16	−81 10	20·52	12 6
73 37	53·50	86 30	164 36	−86 37	63·78	29 43

Principal Incidence $=56°\ 40'$. Coeff. of Refraction $=1{\cdot}5204$.
Circular Limit $=89°\ 5'$. Coeff. of Reflexion $=0{\cdot}0160$.

TABLE XXIX.—Quartz (*a*). (October 15, 1855.)

Compensator $=47{\cdot}12=90°$. Red Sunlight.

Polarizer.	Analyser.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
89° 30′	64° 0′	2·050	1° 1′
89 0	48 0	1·110	1 7
88 0	28 0	1·881	1 4
85 0	11 10	5·066	1 0
			Mean = 1° 3′ 0″

Principal Incidence $=56°\ 40'$. Coeff. of Refraction $=1{\cdot}5204$.
Circular Limit $=88°\ 51'$. Coeff. of Reflexion $=0{\cdot}0200$.

Hence we obtain

TABLE XXX.—Constants of Quartz (a).

No.	Principal Incidence.	Circular Limit.	Coefficient of Refraction.	Coefficient of Reflexion.
XXVIII.	56° 40′	89° 5′	1·5204	0·0160
XXIX.	56 40	88 51	1·5204	0·0200
Means	56 40	88 58	1·5204	0·0180

VII. QUARTZ (b). *Natural surface. Plane of incidence contains optical axis.*

TABLE XXXI.—Quartz (b). (October 16, 1855.)

Azimuth of Polarizer $=88°$. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	φ.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
33° 37′	39·43	86° 30′	0° 0′	+86° 30′	∞	29° 43′
43 37	39·43	84 30	0 0	+84 30	∞	19 56
53 37	39·90	66 0	5 30	+66 4	27·85	4 29
55 37	40·72	45 15	15 5	+45 16	5·57	2 1
56 7	41·82	36 30	27 57	+35 27	4·22	1 29
56 37	43·87	23 30	51 57	+16 44	3·15	0 52
57 7	48·14	27 30	101 54	− 8 12	1·99	1 2
57 37	50·97	39 0	135 0	−36 39	2·47	1 37
63 37	53·85	79 30	168 42	−82 34	7·17	10 40
73 37	54·27	86 50	173 37	−86 51	176·20	32 15

Principal Incidence $=56°\ 57'$. Coeff. of Refraction $=1·5369$.
Circular Limit $=89°\ 8'$. Coeff. of Reflexion $=0·0151$.

TABLE XXXII.—Quartz (b). (October 16, 1855.)

Compensator $=47·12=90°$. Red Sunlight.

Polarizer.	Analyser.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
89° 30′	44° 30′	1·017	0° 30′
89 0	41 10	1·143	0 52
88 0	26 20	2·020	0 59
85 0	13 20	4·219	1 11
			Mean $=0°\ 53'\ 0''$

Principal Incidence $=56°\ 52'$. Coeff. of Refraction $=1·5320$.
Circular Limit $=89°\ 34'$. Coeff. of Reflexion $=0·0076$.

From which we obtain

Table XXXIII.—Constants of Quartz (b).

No.	Principal Incidence.	Circular Limit.	Coefficient of Refraction.	Coefficient of Reflexion.
XXXI.	56° 57′	89° 8′	1·5369	0·0151
XXXII.	56 52	89 34	1·5320	0·0076
Means	56° 54′ 30″	89 21	1·5344	0·0108

The following experiments were made on the metallic bodies.

VIII. Speculum Metal.

Table XXXIV.—Speculum Metal. (July 29, 1854.)
Azimuth of Polarizer =45°. Red Lamplight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	φ.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
35° 7′	40·25	42° 30′	9° 42′	+42° 27′	10·06	42° 30′
44 7	40·73	42 45	15 12	+42 40	7·42	42 45
49 7	41·19	41 30	20 35	+41 16	5·58	41 30
51 7	41·46	40 50	23 45	+40 27	4·78	40 50
53 7	41·62	40 20	25 36	+39 50	4·45	40 20
55 7	41·89	40 3	28 46	+39 22	3·96	40 3
57 7	42·10	39 20	31 14	+38 24	3·65	39 20
59 7	42·63	39 6	37 26	+37 38	3·03	39 6
61 7	43·06	39 20	42 28	+37 24	2·64	39 20
64 7	43·68	38 50	49 42	+35 40	2·24	38 50
69 7	44·96	37 45	64 42	+29 25	1·69	37 45
74 7	46·90	37 45	87 24	+ 4 58	1·29	37 45

Principal Incidence =75° 27′ (?). Circular Limit =52° 15′ (?).

In this experiment the angle of incidence was not made at any time equal to the principal incidence.

Table XXXV.—Speculum Metal. (August 30, 1855.)
Azimuth of Polarizer =50°. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	φ.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
33° 37′	40·00	47° 15′	6° 40′	+47° 16′	16·48	42° 53′
43 37	40·95	45 40	17 47	+45 42	6·40	40 39
53 37	41·80	43 0	27 44	+42 45	4·13	38 3
63 37	43·34	40 15	45 44	+38 16	2·42	35 24
68 37	44·60	39 40	60 29	+34 32	1·77	34 50
73 37	46·26	39 0	79 59	+19 38	1·32	34 12
74 37	46·40	39 32	81 33	+18 38	1·27	34 42
75 37	46·56	40 15	83 18	+17 26	1·23	35 24
76 37	47·03	39 20	87 30	+ 6 8	1·23	34 40
77 37	47·50	39 43	94 25	−11 13	1·22	34 53
78 37	48·00	39 45	100 16	−21 56	1·29	34 55
83 37	50·85	43 0	133 36	−42 6	2·34	38 2
88 37	53·50	49 45	164 37	−49 55	7·65	44 45

Principal Incidence =75° 51′. Coeff. of Refraction =3·9665.
Circular Limit =55° 48′. Coeff. of Reflexion =0·6796.

TABLE XXXVI.—Speculum Metal. (August 28, 1855.)

Azimuth of Polarizer =60°. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	φ.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
33° 37′	40·17	57° 30′	8° 39′	+57° 38′	14·20	42° 11′
43 37	40·76	55 15	15 33	+55 36	7·89	39 46
53 37	41·76	53 30	27 15	+54 30	4·31	37 58
63 37	43·40	52 10	46 27	+55 10	2·44	36 38
68 37	44·60	51 0	60 29	+56 40	1·79	35 29
73 37	46·13	49 30	78 23	+64 6	1·29	34 3
74 37	46·30	49 30	80 22	+66 43	1·26	34 3
75 37	46·55	48 45	83 18	+69 14	1·19	33 22
76 37	46·91	49 15	87 30	+81 52	1·17	33 49
77 37	47·36	48 55	92 47	−80 17	1·16	33 31
78 37	48·25	51 0	103 11	−66 30	1·37	35 29
83 37	50·86	54 30	133 43	−58 14	2·53	38 59
88 37	54·00	60 0	170 28	−60 10	14·13	45 0

Principal Incidence =75° 57′. Coeff. of Refraction =3·9959.
Circular Limit =56° 38′. Coeff. of Reflexion =0·6585.

In this Table, the polarizer having been set at an angle exceeding the circular limit, the axis major of the ellipse passes through 90° at the principal incidence, and behaves exactly as in the transparent bodies.

TABLE XXXVII.—Speculum Metal. (June 29, 1855.)

Azimuth of Polarizer =80°. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	φ.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
33° 37′	40·14	80° 0′	8° 18′	+80° 6′	39·78	45° 0′
43 37	40·97	79 15	18 1	+79 44	17·57	42 53
53 37	41·81	77 30	27 50	+78 48	10·02	38 30
63 37	43·28	76 30	45 2	+80 6	6·06	36 18
68 37	44·44	76 0	58 36	+82 16	4·78	35 16
73 37	46·00	76 0	76 52	+86 33	4·13	35 16
74 37	46·32	75 47	80 36	+87 28	4·01	34 50
75 37	46·57	75 50	83 32	+88 16	3·99	34 56
76 37	46·94	76 0	87 52	+89 26	4·01	35 16
77 37	47·21	75 55	91 1	−89 44	3·99	35 6
78 37	48·15	76 30	102 1	−86 58	4·27	36 18
81 37	49·38	77 30	116 24	−84 9	5·45	38 30
83 37	50·20	77 45	126 0	−82 30	5·89	39 5
85 37	51·30	79 20	128 53	−81 48	8·24	43 7
88 37	54·19	80 0	172 41	−80 4	47·68	45 0

Principal Incidence =76° 7′. Coeff. of Refraction =4·0458.
Circular Limit =55° 10′. Coeff. of Reflexion =0·6959.

TABLE XXXVIII.—Speculum Metal (fresh polished with rouge). (Sept. 11, 1855.)

Compensator $=47{\cdot}12=90°$. Red Sunlight.

Polarizer.	Analyser.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
80°	75° 40′	3·732	34° 37′
70	62 45	1·942	35 15
60	49 45	1·181	34 18
50	38 0	1·279	33 15
40	29 30	1·767	33 57
30	21 45	2·506	34 39
20	14 10	3·961	34 43
10	7 30	7·596	36 45
			Mean = 34° 41′ 7″

Principal Incidence $=78°$ 7′.　　Coeff. of Refraction $=4{\cdot}7522$.
Circular Limit $=55°$ 19′.　　Coeff. of Reflexion $=0{\cdot}6920$.

This experiment shows that the fresh polishing of the surface affected the coefficient of refraction more than the coefficient of reflexion, on which the elliptic polarization altogether depends.

The angle $\tan^{-1}\left(\frac{J}{I}\right)$ is not constant, but attains a minimum at the circular limit.

Additional direct experiments with speculum metal, such as setting the compensator at 90°, making the incidence 76°, setting the analyser at 45°, and then determining the azimuth of the polarizer, gave for the circular limit 54° 45′.

Combining all together, we find

TABLE XXXIX.—Constants of Speculum Metal.

No.	Principal Incidence.	Circular Limit.	Coefficient of Refraction.	Coefficient of Reflexion.
XXXV.	75° 51′	55° 48′	3·9665	0·6796
XXXVI.	75 57	56 38	3·9959	0·6585
XXXVII.	76 7	55 10	4·0458	0·6959
XXXVIII.	78 7	55 19	4·7522	0·6920
Direct Ex.	......	54 45	......	0·7067
Means	76 33	55° 32′ 0″	4·1901	0·6865

IX. Silver.

I examined three descriptions of silver,—

(*a*) Fine silver, rolled.
(*b*) Fine silver, cast.
(*c*) Standard silver, rolled.

Table XL.—Silver (*a*). (September 3, 1855.)
Compensator =47·12=90°. Red Sunlight.

Polarizer.	Analyser.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
80°	79° 30′	5·395	43° 34′
70	69 0	2·605	43 29
60	56 40	1·520	41 17
50	46 15	1·044	41 14
40	36 10	1·368	41 3
30	27 15	1·941	41 44
20	18 0	3·077	41 45
10	9 40	5·870	44 1
			Mean=42° 15′ 52″

Principal Incidence =72° 37′. Coeff. of Refraction =3·1942.
Circular Limit =48° 46′. Coeff. of Reflexion =0·8765.

Table XLI.—Fine Silver (*a*) (newly polished). (September 7, 1855.)
Compensator =47·12=90°. Red Sunlight.

Polarizer.	Analyser.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
80°	79° 40′	5·484	44° 2′
70	68 15	2·506	42 23
60	54 45	1·415	39 15
50	46 15	1·045	41 14
45	42 45	1·082	42 45
40	38 0	1·280	42 57
30	27 40	1·907	42 15
20	19 0	2·904	43 25
10	9 20	6·084	42 59
			Mean=43° 28′ 20″

Principal Incidence =71° 37′. Coeff. of Refraction =3·0090.
Circular Limit =48° 13′. Coeff. of Reflexion =0·8936.

Having set the angle of incidence at 72° 37′, the compensator at 47·12=90°, and the analyser at 45°, I found, by trial, the polarizer or circular limit to be 48° 0′.

Table XLII.—Silver (*b*). (September 6, 1855.)

Compensator =47·12=90°. Red Sunlight.

Polarizer.	Analyser.	$\frac{a}{b}$.	$\mathrm{Tan}^{-1}\left(\frac{J}{I}\right)$.
80°	79° 35′	5·439	43° 48′
70	69 5	2·616	43 36
60	58 40	1·642	43 29
50	47 50	1·104	42 49
45	42 45	1·082	42 45
40	37 30	1·303	42 27
30	28 10	1·867	42 51
20	18 40	2·960	42 52
10	9 50	5·769	44 31
			Mean=43° 14′ 13″

Principal Incidence =78° 7′. Coeff. of Refraction =4·7522.
Circular Limit =47° 13′. Coeff. of Reflexion =0·9255.

Table XLIII.—Silver (*c*). (September 7, 1855.)

Compensator =47·12=90°. Red Sunlight.

Polarizer.	Analyser.	$\frac{a}{b}$.	$\mathrm{Tan}^{-1}\left(\frac{J}{I}\right)$.
80°	79° 30′	5·395	43° 34′
70	68 30	2·538	42 44
60	57 30	1·570	42 11
50	47 15	1·082	42 14
45	42 30	1·091	42 30
40	37 0	1·327	41 55
30	28 0	1·881	42 39
20	19 25	2·837	44 5
10	9 50	5·769	44 30
			Mean=42° 55′ 47″

Principal Incidence =78° 22′. Coeff. of Refraction =4·8573.
Circular Limit =47° 38′. Coeff. of Reflexion =0·9120.

By direct experiment, as before described, I found the circular limit to be 46° 45′.

On the day preceding that on which the experiments were made on *Silver* (*c*), I examined it before polishing, when evidently tarnished with sulphuret, and found

Principal Incidence =67° 37′. Coeff. of Refraction =2·4282.
Circular Limit =52° 30′. Coeff. of Reflexion =0·7673.

Combining the preceding results into one Table, we find,

TABLE XLIV.—Constants of Silver.

SILVER (*a*).	Principal Incidence.	Circular Limit.	Coefficient of Refraction.	Coefficient of Reflexion.
XL.	72° 37′	48° 46′	3·1942	0·8765
XLI.	71 37	48 13	3·0090	0·8936
Direct exp.	......	48 0	......	0·9004
Means	72 7	48° 19′ 40″	3·1016	0·8901
SILVER (*b*). XLII.	78 7	47° 13′	4·7522	0·9255
Silver (*c*). XLIII.	78 22	47 38	4·8573	0·9120
Direct exp.	......	46 45	......	0·9407
Means	78 22	47° 11′ 30″	4·8573	0·9263

Note.—In all the experiments on silver, the minimum value of $\tan^{-1}\left(\frac{J}{I}\right)$, corresponding to the circular limit, is apparent, although, if the surface were mathematically smooth, it ought to be constant, being a function of the incidence only.

X. GOLD (Standard).

TABLE XLV.—(September 20, 1855.)

Compensator $=47{\cdot}12=90°$. Red Sunlight.

Polarizer.	Analyser.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
80°	79° 45′	5·530	44° 17′
70	68 45	2·571	43 6
60	58 15	1·616	43 0
50	47 0	1·072	42 38
45	42 30	1·091	42 30
40	37 45	1·291	42 42
30	28 0	1·881	42 39
20	19 10	2·876	43 41
10	9 40	5·870	44 1
			Mean=43° 10′ 26″

Principal Incidence $=75°$ 37′. Coeff. of Refraction $=3{\cdot}8994$.
Circular Limit $=47°$ 47′. Coeff. of Reflexion $=0{\cdot}9073$.

The minimum value of $\tan^{-1}\left(\frac{J}{I}\right)$ is here also evident.

XI. Mercury (Distilled).

Table XLVI.—(November 1, 1860.)

Compensator $=47{\cdot}12=90°$. Red Lamplight.

Polarizer.	Analyser.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
80°	76° 0′	4·011	35° 16′
70	62 42	1·937	35 11
60	51 35	1·260	36 3
50	41 0	1·150	36 6
40	32 2	1·598	36 43
30	23 25	2·309	36 53
20	14 49	3·780	36 1
10	7 19	7·788	36 4
0	0 0	∞	
			Mean=36° 2′ 7″

Principal Incidence $=81°$ $4'$. Coeff. of Refraction $=6{\cdot}3616$.
Circular Limit $=53°$ $46'$. Coeff. of Reflexion $=0{\cdot}7328$.

By a direct experiment, I obtained, as before described,

Circular Limit $=53°$ $52'$. Coeff. of Reflexion $=0{\cdot}7301$.

The value of $\tan^{-1}\left(\frac{J}{I}\right)$ appears to be constant in mercury: can this be due to its being a liquid?

XII. Platinum.

Table XLVII.—(September 21, 1855.)

Compensator $=47{\cdot}12=90°$. Red Sunlight.

Polarizer.	Analyser.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
80°	76° 10′	4·061	35° 36′
70	63 0	1·962	35 32
60	52 15	1·291	36 43
50	40 10	1·185	35 19
40	32 0	1·600	36 41
30	22 15	2·444	35 19
20	14 45	2·798	35 53
10	8 0	7·115	38 33
			Mean=36° 12′ 0″

Principal Incidence $=76°$ $37'$. Coeff. of Refraction $=4{\cdot}2030$.
Circular Limit $=54°$ $0'$. Coeff. of Reflexion $=0{\cdot}7265$.

XIII. Palladium.

Table XLVIII.—(September 21, 1855.)

Compensator =47·12=90°. Red Sunlight.

Polarizer.	Analyser.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
80°	75° 15′	3·798	33° 49′
70	65 40	2·211	38 50
60	50 10	1·199	34 41
50	40 15	1·181	35 23
40	29 30	1·631	33 59
30	22 50	2·375	36 6
20	15 0	3·732	36 21
10	8 0	7·115	38 33
			Mean=35° 57′ 45″

Principal Incidence =77° 37′. Coeff. of Refraction =4·5546.
Circular Limit =54° 47′. Coeff. of Reflexion =0·7058.

XIV. Copper.

Table XLIX.—Copper. (October 6, 1857.)

Azimuth of Polarizer =46° 15′. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	φ.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
63° 30′	44·96	42° 40′	64° 42′	+39° 35′	1·589	41° 25′
68 30	46·19	42 30	79 0	+32 41	1·236	41 15
69 30	46·49	42 45	82 36	+29 17	1·164	41 30
70 30	46·77	42 25	85 52	+19 17	1·123	41 10
71 30	47·17	42 16	90 34	− 2 57	1·101	41 5
72 30	47·25	42 20	91 30	− 7 50	1·102	41 5
73 30	47·63	42 15	95 57	−23 33	1·152	41 4
74 30	47·81	42 32	98 4	−29 12	1·180	41 17
75 30	48·54	43 50	106 35	−37 34	1·174	42 35
76 30	48·76	43 1	109 9	−39 2	1·416	41 46
78 30	49·84	43 20	121 48	−41 51	1·804	42 5
83 30	51·74	45 24	144 2	−45 30	3·000	44 9

Principal Incidence =71° 21′. Coeff. of Refraction =2·9629.
Circular Limit =48° 55′. Coeff. of Reflexion =0·8718.

Table L.—Copper. (October 6, 1857.)

Azimuth of Polarizer = 47° 45′. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	φ.	$\frac{a}{b}$.	$\mathrm{Tan}^{-1}\left(\frac{J}{I}\right)$.
63° 30′	44·93	45° 10′	64° 24′	+45° 23′	1·593	42° 25′
68 30	46·17	44 20	78 55	+41 34	1·218	41 35
69 30	46·38	44 25	81 18	+41 10	1·166	41 40
70 30	46·67	42 45	84 42	+24 47	1·129	40 1
71 30	47·15	43 50	90 20	− 7 58	1·043	41 6
72 30	47·68	43 20	96 32	−31 27	1·137	40 36
73 30	47·85	43 20	98 31	−34 16	1·174	40 36
74 30	48·23	43 45	102 58	−39 29	1·261	41 1
75 30	48·45	43 45	105 32	−40 22	1·320	41 1
76 30	48·84	43 45	110 6	−41 23	1·435	41 1
78 30	49·71	44 45	120 17	−44 30	1·732	42 0
83 30	51·85	45 50	145 19	−46 1	3·177	43 5

Principal Incidence = 71° 27′. Coeff. of Refraction = 2·9800.

Circular Limit = 49° 59′. Coeff. of Reflexion = 0·8396.

Table LI.—Copper. (October 6, 1857.)

Azimuth of Polarizer = 55°. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	φ.	$\frac{a}{b}$.	$\mathrm{Tan}^{-1}\left(\frac{J}{I}\right)$.
63° 30′	44·72	50° 15′	61° 39′	+55° 40′	1·734	40° 7′
68 30	45·93	50 15	76 3	+63 46	1·362	40 7
69 30	46·17	50 0	78 51	+66 11	1·301	39 51
70 30	46·68	50 20	84 49	+77 12	1·231	40 11
71 30	46·90	50 7	87 24	+82 57	1·204	39 58
72 30	47·24	50 26	91 22	−86 27	1·226	40 16
73 30	47·59	49 30	95 28	−74 29	1·203	39 21
74 30	47·77	49 28	97 34	−70 1	1·228	39 19
75 30	48·23	49 30	102 57	−62 38	1·320	39 21
76 30	48·67	49 47	108 6	−59 15	1·282	39 38
78 30	49·33	51 20	115 49	−58 39	1·683	41 11
83 30	51·37	53 45	139 42	−56 14	2·896	43 41

Principal Incidence = 71° 6′. Coeff. of Refraction = 2·9207.

Circular Limit = 50° 40′. Coeff. of Reflexion = 0·8194.

TABLE LII.—Copper. (September 21, 1855.)

Compensator =47·12=90°. Red Sunlight.

Polarizer.	Analyser.	$\frac{a}{b}$.	$\mathrm{Tan}^{-1}\left(\frac{J}{I}\right)$.
80°	79° 50′	5·576	44° 31′
70	69 15	2·639	43 51
60	59 30	1·697	44 25
50	48 0	1·110	42 59
45	43 0	1·072	43 0
40	37 30	1·303	42 27
30	28 40	1·829	42 47
20	19 30	2·824	44 13
10	9 50	5·769	44 31
			Mean=43° 38′ 13″

Principal Incidence =73° 37′.
Circular Limit =47° 0′.

Coeff. of Refraction =3·4013.
Coeff. of Reflexion =0·9325.

Combining the preceding results, we obtain

TABLE LIII.—Constants of Copper.

No.	Principal Incidence.	Circular Limit.	Coefficient of Refraction.	Coefficient of Reflexion.
XLIX.	71° 21′	48° 55′	2·9629	0·8718
L.	71 27	49 59	2·9800	0·8396
LI.	71 6	50 40	2·9207	0·8194
LII.	73 37	47 0	3·4013	0·9325
Means	71° 52′ 45″	49° 8′ 30″	3·0662	0·8656

XV. Zinc.

Table LIV.—Zinc. (April 22, 1858.)

Azimuth of Polarizer =53°. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	ϕ.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
63 30	43·71	47 0	50 4	48 7	2·143	38 57
68 30	44·58	46 0	60 15	47 1	1·723	37 58
75 30	46·93	45 45	87 45	61 51	1·048	37 43
76 30	46·98	45 0	88 20	+45 0	1·000	37 0
77 30	47·19	45 30	90 47	−70 56	1·022	37 29
78 30	47·84	45 45	98 23	−50 5	1·161	37 43
79 30	48·49	45 30	106 0	−46 49	1·327	37 29
80 30	48·65	46 0	107 52	−48 15	1·375	37 58
81 30	49·20	46 0	114 12	−47 26	1·548	37 58
83 30	50·47	46 30	129 10	−47 22	2·114	38 27
88 30	54·15	52 0	172 13	−52 4	13·078	43 58

Principal Incidence =77° 11′. Coeff. of Refraction =4·3956.
Circular Limit =53° 0′. Coeff. of Reflexion =0·7535.

Table LV.—Zinc. (May 7, 1858.)

Azimuth of Polarizer =57°. Red Sunlight.

Incidence.	Compensator.	Analyser.	$e'-e-180°$.	ϕ.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
63 30	43·37	53 0	46 6	+56 14	2·485	40 45
68 30	44·43	50 30	58 30	+55 12	1·849	38 14
73 30	45·94	48 30	76 9	+58 35	1·315	36 17
75 30	46·49	49 30	82 36	+70 26	1·227	37 15
76 30	46·95	49 0	87 58	+82 55	1·169	36 46
77 30	47·49	50 0	94 18	−78 29	1·224	37 44
78 30	47·74	50 0	97 13	−72 16	1·244	37 44
79 30	48·34	49 30	104 14	−61 5	1·354	37 15
80 30	48·79	50 30	109 30	−60 6	1·491	38 14
81 30	49·32	50 30	115 42	−57 4	1·658	38 14
83 30	50·17	52 0	125 39	−56 35	2·048	39 44
88 30	54·06	54 0	171 10	−54 5	14·983	41 48

Principal Incidence =76° 49′. Coeff. of Refraction =4·2691.
Circular Limit =53° 29′. Coeff. of Reflexion =0·7404.

Table LVI.—Zinc. (September 20, 1855.)

Compensator =47·12=90°. Red Sunlight.

Polarizer.	Analyser.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
80°	75° 0′	3·732	33° 21′
70	62 0	1·881	34 23
60	49 30	1·171	34 3
50	39 45	1·202	34 54
40	29 15	1·785	33 43
30	22 30	2·414	35 39
20	15 0	3·732	36 21
10	7 30	7·596	36 45
			Mean=34° 53′ 37″

Principal Incidence =78° 7′.
Circular Limit =55° 23′.

Coeff. of Refraction =4·7522.
Coeff. of Reflexion =0·6903.

Combining the preceding results, we obtain the following Table for zinc.

Table LVII.—Constants of Zinc.

No.	Principal Incidence.	Circular Limit.	Coefficient of Refraction.	Coefficient of Reflexion.
LIV.	77° 11′	53° 0′	4·3956	0·7535
LV.	76 49	53 29	4·2691	0·7404
LVI.	78 7	55 23	4·7522	0·6903
Means	77° 22′ 20″	53° 57′ 20″	4·4723	0·7281

XVI. Lead (polished).

Table LVIII. (September 20, 1855.)

Compensator =47·12=90°. Red Sunlight.

Polarizer.	Analyser.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
80°	64° 0′	2·050	19° 52′
70	40 30	1·171	17 16
60	29 45	1·750	18 16
50	22 30	2·414	19 10
40	15 0	3·732	17 43
30	10 0	5·671	16 59
20	7 15	7·861	19 16
10	2 30	22·903	13 54
			Mean=17° 48′ 15″

Principal Incidence =69° 37′.
Circular Limit =71° 55′.

Coeff. of Refraction =2·6913.
Coeff. of Reflexion =0·3265.

XVII. Bismuth.

Table LIX. (September 25, 1855.)

Compensator $=47\cdot12=90°$. Red Sunlight.

Polarizer.	Analyser.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
80°	76° 15′	4·086	35° 43′
70	60 45	1·785	33 1
60	50 25	1·209	34 56
50	39 30	1·213	34 40
40	30 30	1·697	35 4
30	21 0	2·605	33 37
20	14 25	3·890	35 14
10	6 30	8·777	32 52
			Mean $=34°\ 23'\ 22''$

Principal Incidence $=73°\ 37'$.
Circular Limit $=55°\ 2'$.
Coeff. of Refraction $=3\cdot4013$.
Coeff. of Reflexion $=0\cdot6993$.

XVIII. Tin.

Table LX. (September 25, 1855.)

Compensator $=47\cdot12=90°$. Red Sunlight.

Polarizer.	Analyser.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
80°	76° 30′	4·165	36° 18′
70	64 10	2·065	36 56
60	52 35	1·307	37 2
50	40 30	1·171	35 38
40	32 0	1·600	36 40
30	22 30	2·414	35 40
20	15 20	3·647	36 59
10	8 10	6·968	39 8
			Mean $=36°\ 47'\ 37''$

Principal Incidence $=75°\ 7'$.
Circular Limit $=53°\ 43'$.
Coeff. of Refraction $=3\cdot7627$.
Coeff. of Reflexion $=0\cdot7341$.

XIX. Iron.

Table LXI.—Hard Steel. (September 29, 1855.)

Compensator =47·12=90°. Red Sunlight.

Polarizer.	Analyser.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
80°	72° 15′	3·124	28° 51′
70	55 30	1·455	27 54
60	42 35	1·088	27 57
50	33 0	1·540	28 36
40	24 35	2·186	28 36
30	17 30	3·171	28 39
20	11 50	4·773	29 57
10	6 15	9·131	31 51
			Mean = 29° 2′ 37″

Principal Incidence =78° 7′. Coeff. of Refraction =4·7522.
Circular Limit =61° 52′. Coeff. of Reflexion =0·5347.

Table LXII.—Soft Steel. (September 29, 1855.)

Compensator =47·12=90°. Red Sunlight.

Polarizer.	Analyser.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
80°	70° 45′	2·863	26° 48′
70	55 50	1·473	28 12
60	41 45	1·120	27 16
50	31 45	1·616	27 27
40	23 30	2·300	27 24
30	17 50	3·108	29 7
20	11 20	4·989	28 50
10	6 0	9·514	30 48
			Mean = 28° 14′ 0″

Principal Incidence =77° 7′. Coeff. of Refraction =4·3721.
Circular Limit =63° 13′. Coeff. of Reflexion =0·5048.

Swedish Iron (cut perpendicular to the grain).

TABLE LXIII. (September 29, 1855.)

Compensator =47·12=90°. Red Sunlight.

Polarizer.	Analyser.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
80°	71° 35′	3·003	27° 54′
70	55 10	1·437	27 37
60	40 45	1·160	26 27
50	32 0	1·600	27 40
40	22 45	2·385	26 33
30	17 0	3·271	27 54
20	11 0	5·144	28 6
10	5 30	10·385	28 38
			Mean=27° 36′ 7″

Principal Incidence =76° 7′.
Circular Limit =62° 57′.

Coeff. of Refraction =4·0458.
Coeff. of Reflexion =0·5106.

Swedish Iron (cut parallel to the grain).

TABLE LXIV. (September 29, 1855.)

Polarizer.	Analyser.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
80°	71° 45′	3·032	28° 8′
70	55 20	1·446	27 46
60	41 40	1·124	27 12
50	32 0	1·600	27 40
40	24 0	2·246	27 57
30	17 30	3·171	28 39
20	11 0	5·144	28 6
10	5 35	10·229	29 0
			Mean=28° 3′ 30″

Principal Incidence =76° 7′.
Circular Limit =62° 26′.

Coeff. of Refraction =4·0458.
Coeff. of Reflexion =0·5220.

Combining the preceding results, we find

TABLE LXV.—Constants of Steel and Iron.

No.	Principal Incidence.	Circular Limit.	Coefficient of Refraction.	Coefficient of Reflexion.
LXI. Hard steel.	78° 7′	61° 52′	4·7522	0·5347
LXII. Soft steel.	77 7	63 13	4·3721	0·5048
LXIII. Iron (*a*).	76 7	62 57	4·0458	0·5106
LXIV. Iron (*b*).	76 7	62 26	4·0458	0·5220

XX. Aluminium.

Table LXVI. (May 10, 1856.)

Compensator =47·12=90°. Red Sunlight.

Polarizer.	Analyser.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
80°	75° 45′	3·937	34° 46′
70	60 30	1·767	32 45
60	48 0	1·110	32 40
50	37 30	1·303	32 47
40	28 15	1·861	32 38
30	20 30	2·674	32 56
20	13 45	4·086	33 55
10	7 10	7·953	35 30
			Mean=33° 29′ 37″

Principal Incidence =77° 7′. Coeff. of Refraction =4·3721.
Circular Limit =57° 9′. Coeff. of Reflexion =0·6457.

By a direct experiment I found the circular limit to be 57° 15′.

XXI. Alloys of Copper and Zinc.

The following experiments were made on fourteen alloys of copper and zinc prepared by Mr. Robert Mallet, in atomic proportions, as follow:—

No. 1 . . .	10 Cu+ Zn
No. 2 . . .	9 Cu+ Zn
No. 3 . . .	8 Cu+ Zn
No. 4 . . .	7 Cu+ Zn
No. 5 . . .	6 Cu+ Zn
No. 6 . . .	5 Cu+ Zn
No. 7 . . .	4 Cu+ Zn
No. 8 . . .	3 Cu+ Zn
No. 9 . . .	2 Cu+ Zn
No. 10 . . .	Cu+ Zn
No. 11 . . .	Cu+2Zn
No. 12 . . .	Cu+3Zn
No. 13 . . .	Cu+4Zn
No. 14 . . .	Cu+5Zn

The chemical and physical properties of these alloys are fully described by Mr. Mallet in his "Report on the Action of Air and Water upon Iron" to the British Association for the Advancement of Science for the year 1840, p. 306.

In all the experiments red sunlight was used, and the compensator was placed at 47·12=90°.

TABLE LXVII.—Alloys of Copper and Zinc, No. 1. (September 16, 1856.)

Polarizer.	Analyser.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
80°	79° 0′	5·144	42° 13′
70	66 45	3·237	40 16
60	56 10	1·492	40 45
50	45 30	1·017	40 30
40	35 30	1·402	40 22
30	27 0	1·962	41 26
20	18 15	3·032	42 11
10	9 25	6·029	43 15
			Mean=41° 22′ 15″

Principal Incidence =72° 5′. Coeff. of Refraction =3·0930.
Circular Limit =49° 32′. Coeff. of Reflexion =0·8531.

TABLE LXVIII.—Alloys of Copper and Zinc, No. 2. (September 16, 1856.)

Polarizer.	Analyser.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
80°	79° 40′	5·484	44° 3′
70	67 35	2·424	41 25
60	58 0	1·600	42 44
50	45 35	1·020	40 35
40	35 25	1·406	40 17
30	27 30	1·921	42 2
20	17 40	3·140	41 11
10	9 30	5·976	43 30
			Mean=41° 58′ 22″

Principal Incidence =72° 15′. Coeff. of Refraction =3·1240.
Circular Limit =49° 32′. Coeff. of Reflexion =0·8531.

Table LXIX.—Alloys of Copper and Zinc, No. 3. (September 18, 1856.)

Polarizer.	Analyser.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
80°	79° 10′	5·225	44° 20′
70	67 20	2·394	41 4
60	54 0	1·376	38 28
50	46 0	1·035	40 59
40	34 50	1·437	39 40
30	27 15	1·941	41 44
20	17 35	3·155	41 2
10	9 15	6·140	42 43
			Mean=41° 2′ 30″

Principal Incidence =73°· 10′. Coeff. of Refraction =3·3052.
Circular Limit =49° 6′. Coeff. of Reflexion =0·8662.

Table LXX.—Alloys of Copper and Zinc, No. 4. (September 18, 1856.)

Polarizer.	Analyser.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
80°	78° 30′	4·915	40° 55′
70	67 45	2·444	41 40
60	57 0	1·540	41 38
50	46 0	1·035	40 59
40	35 30	1·402	40 22
30	27 0	1·962	41 26
20	18 0	3·077	41 45
10	8 50	6·435	41 23
			Mean = 41° 16′ 0″

Principal Incidence =73° 8′. Coeff. of Refraction =3·2983.
Circular Limit =49° 3′. Coeff. of Reflexion =0·8677.

Table LXXI.—Alloys of Copper and Zinc, No. 5. (September 18, 1856.)

Polarizer.	Analyser.	$\frac{a}{b}$.	$\tan^{-1}\left(\frac{J}{I}\right)$.
80°	79° 10′	5·225	42° 40′
70	67 0	2·356	40 37
60	56 15	1·496	40 50
50	45 55	1·032	40 55
40	35 52	1·383	40 45
30	26 50	1·977	41 13
20	18 15	3·032	42 11
10	9 45	5·819	44 16
			Mean = 41° 40′ 52″

Principal Incidence =74° 5′. Coeff. of Refraction =3·5066.
Circular Limit =49° 5′. Coeff. of Reflexion =0·8667.

TABLE LXXII.—Alloys of Copper and Zinc, No. 6. (September 19, 1856.)

Polarizer.	Analyser.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
80°	79° 25′	5·352	43° 20′
70	68 30	2·538	42 44
60	57 55	1·595	42 39
50	47 40	1·098	42 39
40	36 30	1·351	41 24
30	28 15	1·861	42 57
20	18 12	3·041	42 5
10	9 45	5·819	44 16
			Mean = 42° 45′ 30″

Principal Incidence = 74° 8′. Coeff. of Refraction = 3·5183.
Circular Limit = 47° 37′. Coeff. of Reflexion = 0·9126.

TABLE LXXIII.—Alloys of Copper and Zinc, No. 7. (September 19, 1856.)

Polarizer.	Analyser.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
80°	79° 10′	5·225	42° 39′
70	67 30	2·414	41 18
60	57 0	1·540	41 39
50	45 40	1·023	40 39
40	35 0	1·428	39 51
30	26 10	2·035	40 24
20	17 45	3·124	41 20
10	9 25	6·029	43 15
			Mean = 41° 48′ 7″

Principal Incidence = 73° 16′. Coeff. of Refraction = 3·3261.
Circular Limit = 49° 23′. Coeff. of Reflexion = 0·8576.

TABLE LXXIV.—Alloys of Copper and Zinc, No. 8. (July 13, 1857.)

Polarizer.	Analyser.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
80°	77° 30′	4·511	38° 30′
70	65 10	2·161	38 11
60	55 40	1·464	40 13
50	43 50	1·041	38 51
40	33 5	1·535	37 49
30	24 40	2·177	38 30
20	15 40	3·565	37 37
10	8 20	6·827	39 43
			Mean = 38° 40′ 30″

Principal Incidence = 73° 12′. Coeff. of Refraction = 3·3121.
Circular Limit = 50° 53′. Coeff. of Reflexion = 0·8132.

TABLE LXXV.—Alloys of Copper and Zinc, No. 9. (October 5, 1857.)

Polarizer.	Analyser.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
80°	79° 0′	5·144	42° 13′
70	67 30	2·414	41 18
60	54 55	1·424	39 25
50	46 20	1·047	41 19
40	35 35	1·397	40 27
30	26 0	2·050	40 11
20	17 0	3·271	40 2
10	8 40	6·560	40 50
			Mean = 40° 43′ 7″

Principal Incidence = 72° 18′. Coeff. of Refraction = 3·1334.
Circular Limit = 48° 46′. Coeff. of Reflexion = 0·8764.

TABLE LXXVI.—Alloys of Copper and Zinc, No. 10. (October 5, 1857.)

Polarizer.	Analyser.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
80°	79° 0′	5·144	42° 13′
70	65 0	2·144	37 58
60	54 20	1·393	38 49
50	43 45	1·044	38 46
40	34 35	1·450	39 35
30	25 15	2·120	39 15
20	16 30	3·375	39 9
10	8 45	6·497	41 7
			Mean = 39° 36′ 30″

Principal Incidence = 72° 15′. Coeff. of Refraction = 3·1240.
Circular Limit = 51° 11′. Coeff. of Reflexion = 0·8045.

TABLE LXXVII.—Alloys of Copper and Zinc, No. 11. (October 5, 1857.)

Polarizer.	Analyser.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
80°	78° 30′	4·915	40° 55′
70	65 30	2·194	38 37
60	54 0	1·376	38 28
50	43 0	1·072	38 3
40	33 30	1·511	38 16
30	24 30	2·194	38 17
20	16 15	3·431	38 41
10	8 35	6·625	40 34
			Mean = 38° 58′ 52″

Principal Incidence = 72° 15′. Coeff. of Refraction = 3·1240.
Circular Limit = 51° 49′. Coeff. of Reflexion = 0·7864.

TABLE LXXVIII.—Alloys of Copper and Zinc, No. 12. (October 5, 1857.)

Polarizer.	Analyser.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
80°	74° 40′	3·647	32° 45′
70	58 45	1·648	30 57
60	48 0	1·110	32 40
50	37 45	1·291	33 1
40	28 10	1·867	31 57
30	20 0	2·747	32 14
20	13 20	4·219	33 4
10	6 30	8·776	32 56
			Mean = 32° 26′ 45″

Principal Incidence = 76° 7′. Coeff. of Refraction = 4·0458.
Circular Limit = 57° 5′. Coeff. of Reflexion = 0·6473.

TABLE LXXIX.—Alloys of Copper and Zinc, No. 13. (October 6, 1857.)

Polarizer.	Analyser.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
80°	76° 15′	4·086	35° 46′
70	60 50	1·792	33 7
60	49 45	1·181	34 18
50	39 10	1·227	34 21
40	29 15	1·785	33 43
30	20 40	2·651	33 10
20	13 30	4·165	33 25
10	6 40	8·555	33 32
			Mean = 33° 55′ 15″

Principal Incidence = 73° 52′. Coeff. of Refraction = 3·4570.
Circular Limit = 55° 31′. Coeff. of Reflexion = 0·6868.

TABLE LXXX.—Alloys of Copper and Zinc, No. 14. (October 6, 1857.)

Polarizer.	Analyser.	$\frac{a}{b}$.	$\text{Tan}^{-1}\left(\frac{J}{I}\right)$.
80°	75° 45′	3·937	34° 46′
70	61 45	1·861	34 7
60	48 40	1·136	33 17
50	39 0	1·235	34 12
40	28 40	1·829	33 5
30	20 50	2·628	33 23
20	13 55	4·036	34 15
10	7 10	7·953	35 30
			Mean = 34° 4′ 22″

Principal Incidence = 76° 0′. Coeff. of Refraction = 4·0108.
Circular Limit = 56° 12′. Coeff. of Reflexion = 0·6694.

The alloys from 1 to 11 are all yellowish, and from 12 to 14 are whitish.

The following Table shows that the Coefficients of Refraction from 1 to 11 increase gradually, reaching a maximum at No. 6 (5 Cu+Zn), and then diminish to No. 11, in passing from which to No. 12 the coefficient suddenly increases. The Coefficient of Reflexion follows an order somewhat similar, but suddenly decreases in passing from 11 to 12, which is the limit at which the zinc begins to preponderate over the copper, in producing the optical properties of the alloy.

In Plate VIII. fig. B, I have tabulated the coefficients of refraction and reflexion of the alloys of copper and zinc, showing the progression of these constants, as just described.

TABLE LXXXI.—Optical Constants of all the Substances examined.

Substance.	Principal Incidence.	Circular Limit.	Coefficient of Refraction.	Coefficient of Reflexion.	Refractive Index.
(A.) *Transparent.*					
I. Munich Glass (*a*)	55° 0′ 37″	85° 32′ 30″	1·4287	0·0780	1·6227
II. Munich Glass (*b*)	54 40 48	89 53 24	1·4113	0·0019	1·5244
III. Paris Glass	56 8 30	89 23 0	1·4905	0.0107	1·5100
IV. Fluor-Spar	54 46 0	89 41 30	1·4158	0·0053	
V. Glass of Antimony	58 48 40	88 51 40	1·6519	0·0199	
VI. Quartz (*a*)	56 40 0	88 58 0	1·5204	0·0180	
VII. Quartz (*b*)	56 54 30	89 21 0	1·5344	0·0108	
(B.) *Metals.*					
VIII. Speculum	76 33 0	55 32 0	4·1901	0·6865	
IX. Silver (*a*)	72 7 0	48 19 40	3·1016	0·8901	
— Silver (*b*)	78 7 0	47 13 0	4·7522	0·9255	
— Silver (*c*)	78 22 0	47 11 30	4·8573	0·9263	
X. Gold	75 37 0	47 47 0	3·8994	0·9073	
XI. Mercury	81 4 0	53 49 0	6·3616	0·7315	
XII. Platinum	76 37 0	54 0 0	4·2030	0·7265	
XIII. Palladium	77 37 0	54 47 0	4·5546	0·7058	
XIV. Copper	71 52 45	49 8 30	3·0662	0·8656	
XV. Zinc	77 22 20	53 57 20	4·4723	0·7281	
XVI. Lead	69 37 0	71 55 0	2·6913	0·3265	
XVII. Bismuth	73 37 0	55 2 0	3·4013	0·6993	
XVIII. Tin	75 7 0	53 43 0	3·7627	0·7341	
XIX. Iron	76 7 0	62 41 30	4·0458	0·5163	
— Steel	77 37 0	62 32 30	4·5621	0·5197	
XX. Aluminium	77 7 0	57 9 0	4·3721	0·6457	
XXI. Alloys of Copper and Zinc:—					
No. 1	72 5 0	49 32 0	3·0930	0·8531	
No. 2	72 15 0	49 32 0	3·1240	0·8531	
No. 3	73 10 0	49 6 0	3·3052	0·8662	
No. 4	73 8 0	49 3 0	3·2983	0·8677	
No. 5	74 5 0	49 5 0	3·5066	0·8667	
No. 6	74 8 0	47 37 0	3·5183	0·9126	
No. 7	73 16 0	49 23 0	3·3261	0·8576	
No. 8	73 12 0	50 53 0	3·3121	0·8132	
No. 9	72 18 0	48 46 0	3·1334	0·8764	
No. 10	72 15 0	51 11 0	3·1240	0·8045	
No. 11	72 15 0	51 49 0	3·1240	0·7864	
No. 12	76 7 0	57 5 0	4·0458	0·6473	
No. 13	73 52 0	55 31 0	3·4570	0·6868	
No. 14	76 0 0	56 12 0	4·0108	0·6694	

In the preceding Table there are twelve pure metals; if we arrange these in two Tables, according to the magnitude of the Coefficients of Refraction and Reflexion, we obtain the following.

Table LXXXII.—Coefficient of Refraction of pure Metals.

Metal.	Coefficient of Refraction.
I. Mercury	6·3616
II. Silver	4·8047
III. Palladium	4·5546
IV. Zinc	4·4723
V. Aluminium	4·3721
VI. Iron	4·3039
VII. Platinum	4·2030
VIII. Gold	3·8994
IX. Tin	3·7627
X. Bismuth	3·4013
XI. Copper	3·0662
XII. Lead	2·6913

Table LXXXIII.—Coefficient of Reflexion of pure Metals.

Metal.	Coefficient of Reflexion.
I. Silver	0·9259
II. Gold	0·9073
III. Copper	0·8656
IV. Tin	0·7341
V. Mercury	0·7315
VI. Zinc	0·7281
VII. Platinum	0·7265
VIII. Palladium	0·7058
IX. Bismuth	0·6993
X. Aluminium	0·6457
XI. Iron	0·5180
XII. Lead	0·3265

The *brilliancy* of a metallic surface depends on the coefficient of refraction, and there is, doubtless, some sensible quality, not yet clearly defined, which corresponds to the coefficient of reflexion, which indicates the power of the surface to form elliptically polarized light from incident plane-polarized light. This quality might be provisionally named *lustre*.

It is very remarkable that gold, silver, and copper, which from time immemorial have pleased the eye of man, and been used as coins, should head the list of bodies possessing a high coefficient of reflexion. Mercury, which has so brilliant a surface, and therefore heads the list in Table LXXXII., occupies a comparatively low place in Table LXXXIII., probably owing to its being a liquid, and its surface, therefore, in a less favourable condition than that of a solid for imparting elliptic polarization to an incident beam.

M. Jamin has examined optically several of the substances mentioned in the preceding

Tables—the metallic bodies by the methods of equal intensities and multiple reflexions, and the transparent bodies by the method employed in this paper, and originally used by him.

I have deduced from his original observations, the optical constants of the substances common to him and myself, and have recorded them for the purpose of comparison, in the two following Tables, LXXXIV. and LXXXV.*

TABLE LXXXIV.—Optical Constants of Metals, deduced from JAMIN's experiments.

Substance.	Principal Incidence.	Circular Limit.	Coefficient of Refraction.	Coefficient of Reflexion.
Steel (1)	76° 0′	59° 6′	4·0108	0·5985
Steel (2)	77 4	61 27	4·3546	0·5441
I. Means	76° 32′ 0″	60° 16′ 30″	4·1827	0·5713
Silver (3)	71 40	54 0	3·0178	0·7265
Silver (4)	75 0	47 1	3·7320	0·9320
II. Means	73° 20′ 0″	50° 30′ 30″	3·3747	0·8292
Zinc (5)	77 0	......	4·3314	......
Zinc (5)	79 13	....	5·2505	....
Zinc (6)	75 11	60 57	3·7804	0·5554
III. Means	77° 8′ 0″	60° 57′ 0″	4·4541	0·5554
Copper (7)	70 9	50 36	2·7700	0·8214
Copper (8)	71 21	53 41	2·9629	0·7350
IV. Means..................	70° 45′ 0″	52° 8′ 30″	2·8664	0·7782
Speculum metal (9)	75 50	56 45	3·9616	0·6556
Speculum metal (10)	......	56 40	......	0·6577
Speculum metal (11)	76 14	53 33	4·0815	0·7386
V. Means	76° 2′ 0″	55° 39′ 20″	4·0215	0·6839
VI. Brass (12)	71 31	52 57	2·9916	0·7549

* These Tables were added during the printing of the paper.

TABLE LXXXV.—Optical Constants of Transparent Bodies, from JAMIN's experiments.

Substance.	Principal Incidence.	Circular Limit.	Coefficient of Refraction.	Coefficient of Reflexion.
I. Glass of Antimony (13)	63° 34′	88° 20′	2·0115	0·0290
II. Quartz (13)	56 50	89 25	1·5301	0·0102
III. Fluor-Spar (13)	55 15	89 31	1·4415	0·0084

(1) Ann. de Chim. et de Phys. (sér. 3) vol. xix. p. 304.

From the two Tables in this page, I find at 75° incidence, I=0·946, and J=0·566, from which it follows that

$$\tan^{-1}\left(\frac{J}{I}\right)=30°\ 54'.$$

(2) Ann. de Chim. et de Phys. (sér. 3) vol. xxii. p. 316 (mean red).

The azimuths given in this and the following page are arcs such that

$$\tan\ (\text{azimuth})\ =k^2$$

$$k=\tan^{-1}\left(\frac{J}{I}\right).$$

From this consideration the coefficient of reflexion is deduced.

(3) Ann. de Chim. et de Phys. (sér. 3) vol. xix. p. 315.

(4) Ann. de Chim. et de Phys. (sér. 3) vol. xxii. p. 316 (mean red).

(5) Ann. de Chim. et de Phys. (sér. 3) vol. xix. p. 320.

(6) Ann. de Chim. et de Phys. (sér. 3) vol. xxii. p. 316 (mean red).

(7) Ann. de Chim. et de Phys. (sér. 3) vol. xix. p. 337. I have calculated the value of the circular limit and coefficient of refraction from the experiment recorded as made with *two* reflexions.

(8) Ann. de Chim. et de Phys. (sér. 3) vol. xxii. p. 317 (red light).

(9) Ann. de Chim. et de Phys. (sér. 3) vol. xix. pp. 305, 306. The ratio of J to I at the principal incidence is found to be, from the Tables of these two pages, as 623 to 950, from which the circular limit is deduced.

(10) Ann. de Chim. et de Phys. (sér. 3) vol. xix. p. 330.

(11) Ann. de Chim. et de Phys. (sér. 3) vol. xxii. p. 316 (red light).

(12) Ann. de Chim. et de Phys. (sér. 3) vol. xxii. p. 317 (red light).

(13) Ann. de Chim. et de Phys. (sér. 3) vol. xxix. p. 303.

Fig. A.

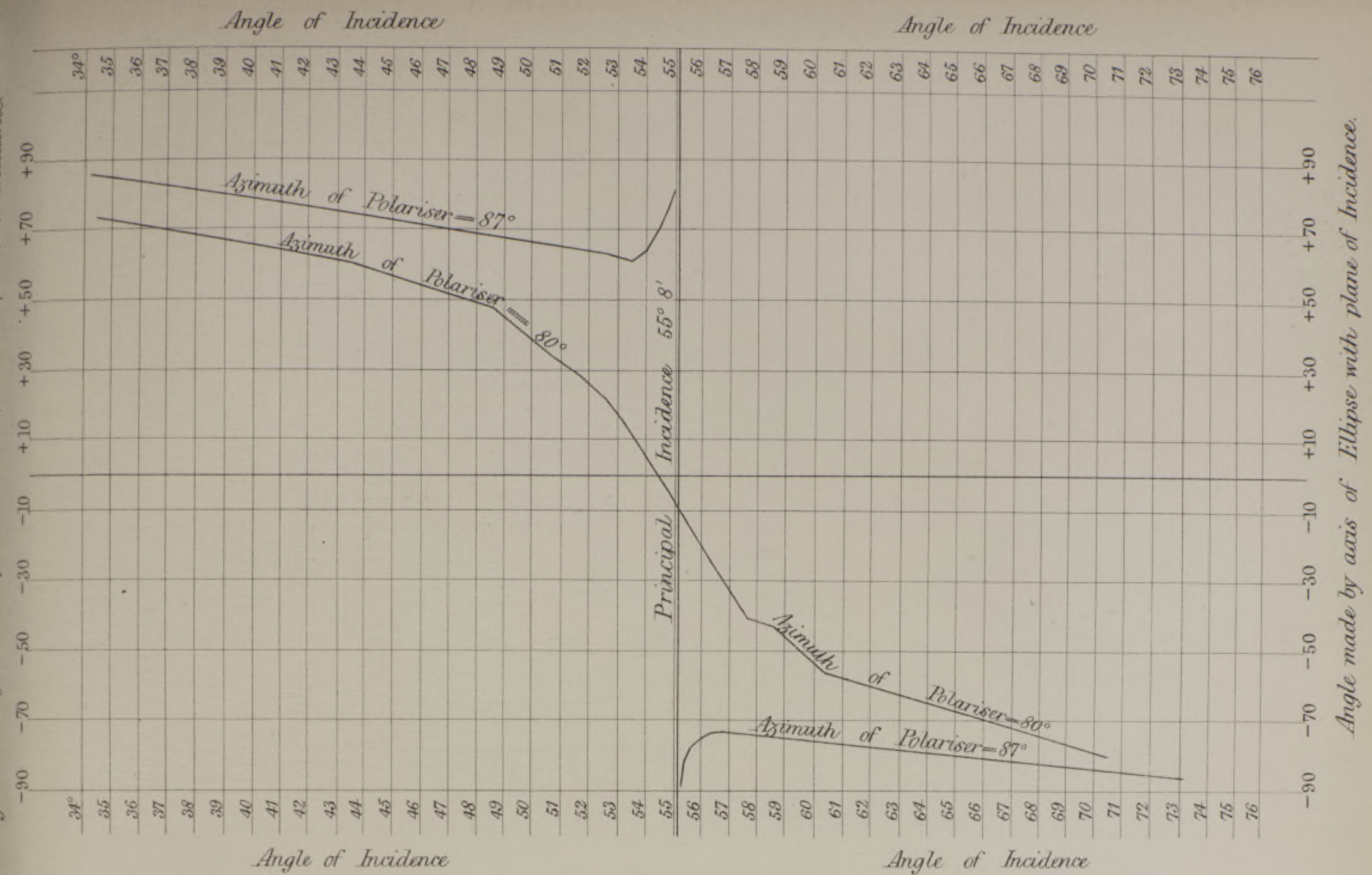
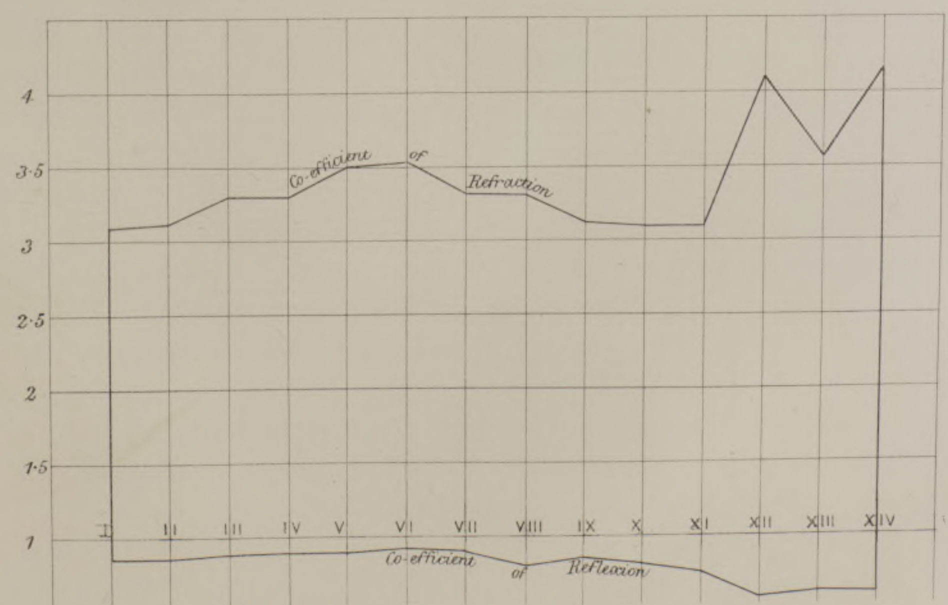


Fig. B.



J. Basire, lith.